BC-TOP-9A
Revision 2
SEPTEMBER 1974

TOPICAL REPORT
DESIGN OF STRUCTURES
FOR MISSILE IMPACT

MASTER

Bechtel Power Corporation
San Francisco, California

TOPICAL REPORT

BC-TOP-9-A

Revision 2

DESIGN OF STRUCTURES FOR MISSILE IMPACT

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BECHTEL POWER CORPORATION   Issue Date: September 1974

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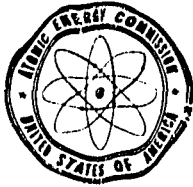
Mr. John V. Morowski
Vice President-Engineering
Bechtel Power Corporation
Fifty Beale Street
San Francisco, California 94119

Dear Mr. Morowski:

The Regulatory staff has completed its review of Bechtel Power
Corporation's Topical Report, BC-TOP-9, Revision 2, dated September
1974 and entitled "Design of Structures for Missile Impact". We
conclude that the design criteria and procedures described by this
report are acceptable to the Regulatory staff and that BC-TOP-9,
Revision 2, is acceptable by reference in applications for construction
permits and operating licenses. A summary of our evaluation is
enclosed.

BC-TOP-9 does not provide all of the pertinent information required
by the Regulatory staff in its review of specific applications.
Therefore, the appropriate supplementary info·.mation identified in
the Regulatory Position of the enclosed Topical Report Evaluation
will have to be provided in individual Safety Analysis Reports.

The staff does not intend to repeat its review of BC-TOP-9, Revision
2, when it appears as a reference in a particular license application.
Should Regulatory criteria or regulations change, such that our
conclusions concerning BC-TOP-9, Revision 2, are invalidated, you
will be notified and given the opportunity to revise and resubmit
your topical report for review, should you so desire.

We request that you reissue BC-TOP-9, Revision 2, dated September
1974 in accordance with the provisions of the "Elements of the
Regulatory Staff Topical Report Review Program" which was forwarded
to you on August 26, 1974. If you have any questions in this
regard, please let us know.

Sincerely,

R. W. Klecker, Technical Coordinator
    for Light Water Reactors Group 1
Directorate of Licensing

Enclosure:
Topical Report Evaluation

Topical Report Evaluation

Report: BC-TOP-9 Rev.2
Report Title: Design of Structures for Missile Impact
Report Date: September 1974
Originating Organization: Bechtel Power Corporation
Reviewed by: Structural Engineering Branch, November 1974

Summary of Report

This report contains the current general procedures and criteria
used by Bechtel Power Corporation for design of nuclear power
plant structures and components against the effects of impact of
missiles. The report covers the evaluation of local effects due to
missiles impacting on both concrete and steel structural elements.
It also covers the procedures used to evaluate the overall structural
response to missile impact loads. Design guidelines related to use
of dynamic capacity increase factors, allowable ductility ratio and
allowable range of steel ratios used in concrete structural elements
are also discussed in the report. Brief discussions of special
problems related to (a) force-time history for automobile crash and
(b) penetration of a missile through a liquid are included as a
part of the report.

The formulae which can be used to predict the penetration resulting
from missile impact are included in the report. The penetration and
perforation formulae assume that the missile strikes the target normal
to the surface, and the axis of the missile is assumed parallel to
the line of flight. These assumptions result in a conservative
estimate of local damage to the target. The formula used to predict
the penetration is the Modified Petry equation, while that for per-
foration and spalling is the Ballistic Research Laboratory formula mod-
ified to allow its use for concrete strength other than 3000 psi
by replacing the constant coefficient 7.8 by $427/\sqrt{f'_c}$. The wall
thicknesses to prevent perforation and spalling are that calculated
using the Ballistic Research Laboratory formula multiplied by factors
of 1.25 and 2.5, respectively. The Ballistic Research Laboratory
formula for steel is used to predict design thickness requirement

for steel targets. The thicknesses of steel targets to prevent perforation are obtained by multiplying 1.25 by the thicknesses for threshold perforation as determined by the BRL formula.

The report discusses both elastic and plastic modes of overall structural response of target subjected to a missile impact. Expressions for (a) velocities of missile and target after impact, (b) strain energy of a target required to stop a missile after impact, (c) target effective mass definition and (d) resistance functions for various target configurations are presented in the report. The overall structural response of a target is determined by equating the available target strain energy to the required strain energy to stop a missile. The resistance function for a structural element is determined using yield-line theory for concentrated loads impacting steel and reinforced concrete beam and slab. The allowable ductility ratios to be used for design are based on the available data from the literature accepted in the engineering practice. However the governing requirement for an overall structural response design con-sideration is that the maximum deflection of the target shall be limited so as not to impair the function of other safety related equipment. Due to the complexity of the impact phenomena, the target effective mass is conservatively derived based on the tests performed on concrete slabs and beams.

The report covers two types of special problems, i.e., determination of an empirical formula for force-time history of automobile crash and an evaluation of a missile velocity as it passes through a liquid.

In deriving the force-time history of an automobile crash under frontal impact, the automobile is considered as a deformable missile and the structure a rigid target. The pertinent equations are based on theoretical considerations backed by experimental data.

The derivation of the velocity of a missile after it has penetrated through a liquid takes into consideration the buoyant force, which is variable during the process of immersion of the missile and constant after the entire missile is immersed in the liquid, and drag force which may be considered as constant for any particular set of conditions. The non-linear, second order, non homogeneous differential equation is transformed into a linear differential equation which is solved by applying pertinent boundary conditions.

For the postulated missiles and their properties as well as for structures, shields and barriers that are required to be designed against effects of missile impact, the report refers to the plant SAR.

Appendix A provides the cross reference between sections of the AEC's Standard SAR format and the sections of BC-TOP-9. Glossary of the report is given in Appendix B. A review of existing design formulas is given in Appendix C whereas Appendix D discusses theoretical derivation for force-time history associated with automobile crash and velocity of a missile penetrating through a liquid. Sample applications of the procedures presented in the report are shown in Appendix E with references and bibliography listed in Appendix F.

Summary of the Regulatory Evaluation

The Structural Engineering Branch of the Directorate of Licensing has reviewed the subject report and its appendices. The procedures covered by this report with the qualifications stated in the following Regulatory Position and augmentation of pertinent information that is referred to and to be provided in the plant SAR are judged to represent the present "state of the art" in the field of design of structures and components against missile impacts. If properly utilized in nuclear power plant structural design work, the procedures and criteria contained in the report should provide

conservative and acceptable bases for design of structural elements against missile impact effects.

Regulatory Position

The design criteria and procedures are acceptable to the Regulatory staff. The report may be referenced in future case applications provided that the following specific information reviewed and accepted by the Regulatory staff is included in individual SAR:

a) Parameters that define the postulated missiles such as striking velocity, weight, missile configurations and impacting area, etc.

b) Structures, shields and barriers that are required to be designed for missiles with their pertinent characteristics.

c) If use of a ductility ratio greater than 10 (i.e., $\mu > 10$) is required to demonstrate design adequacy of structural elements against missile impact, such a usage should be identified in the plant SAR. Information justifying the use of this relatively high ductility value may become necessary for inclusion in the plant SAR. In such a case, the Regulatory staff will request the applicant to provide the information on a case by case basis.

d) The evaluation of punching shear effect due to impact of unconventional missiles, is not included as a part of the overall structural response consideration in the report. The subject should be adequately addressed in individual plant SAR.

ABSTRACT

This report contains methods and procedures for evaluating the effects of missile impact on structures. A means to evaluate the change of velocity of a missile passing through a liquid is also included. Missile impact effects on structures are evaluated in terms of local damage (penetration, perforation, and spalling) and structural response. Empirical formulae are used to evaluate local effects. Structural dynamic principles are used to evaluate structural response.

ACKNOWLEDGMENT

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Section 1

INTRODUCTION

1.1 GENERAL

The design of nuclear power facilities includes the effects of missile impact on structures, systems, and equipment. External building surfaces, interior walls and floors, and special barriers (constructed of concrete and/or steel) that will resist or deflect missiles may be used to protect systems and equipment where necessary.

This report contains methods and preferred procedures to evaluate missile impact on structures and barriers. Missile effects are evaluated in terms of local damage (penetration, perforation, and spalling) and structural response.

Missiles may be generated by an event that is not related to plant operation, or by the failure of plant equipment. The primary sources of missiles, not related to plant operations are debris transported by tornado winds, and falling objects generated by activities near the plant site (such as commercial, industrial, or military activities). Missiles that may result from the failure of equipment generally result from the uncontrolled release of energy and forces from a pressurized system or rotating machinery. Missiles that may result from the failure of equipment are fittings, valve parts, various nuts and bolts, and parts of rotating machinery, etc.

1.2 APPROACH

Determining the effect of missile impact is outlined in the following general steps. However, there are many interactive effects in each step that should be considered in the complete analysis.

- Determine missile characteristics.

- Define target, considering impact in combination with other loads and requirements (preliminary properties).

- Determine local effects of missile on target.

- Determine target characteristics for structural response and stability.

- Determine equivalent target mass during impact.

- Determine structural response.

- Evaluate structural integrity.

- Verify that the maximum deflection does not impair the function of other safety related systems.

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1.3 MISSILE CHARACTERISTICS

Missile parameters required for missile impact analysis include trajectory, mass, velocity, geometry, and deformation characteristics. The geometry should include contact area, projected frontal area and variation of area with respect to length. Deformation characteristics include if the missile will deform or is rigid and if it is ductile or brittle. Missile geometry and deformation characteristics have a significant effect on penetration or perforation of a target. A pointed missile will penetrate deeper into a target than a blunt missile; it will also perforate a thicker target. Deformation of a missile during impact consumes energy, which results in diminished local damage.

Postulated missiles and their properties may vary with each plant and are defined in the Safety Analysis Report (SAR) for nuclear power plants.

1.4 TARGET CHARACTERISTICS

Structures or barriers (targets), providing missile protection, act as energy absorbers. The target absorbs the energy by local damage at the location of impact (i.e. penetration of the missile into the barrier) and by the structural response of the target.

Local damage depends on missile characteristics, target material properties, and structural response. Empirical methods are used to estimate local damage because of the complex phenomena associated with missile impact. The ability of a target to absorb energy by structural response depends on the dynamic properties of the target, support conditions and other imposed loads at the time of impact. Structural dynamic principles are used to estimate the structural response and determine if the target will remain stable during and after missile impact.

Structures, shields and barriers that are required to be designed for a missile are given in the Safety Analysis Reports.

Section 2

LOCAL EFFECTS

Predicting local damage in the impact area includes estimating depth of
penetration, minimum thickness required to prevent perforation, and minimum
thickness required to preclude spalling. The penetration and perforation
formulae in this section assume that the missile strikes the target normal
to the surface, and the axis of the missile is assumed parallel to the line
of flight. These assumptions result in a conservative estimate of local
damage to the target. Appendix C has information on the more common local
effect formula and a discussion of the effects on the penetration for a
missile striking a target at oblique angle.

2.1 REINFORCED CONCRETE TARGETS

2.1.1 PENETRATION

The depth to which a rigid missile will penetrate a reinforced concrete
target of infinite thickness is estimated by the following formula

$$X = 12\ K_p\ A_p\ Log_{10}\left(1 + \frac{V_s^2}{215,000}\right) \qquad (2\text{-}1)$$

where

X = Depth of missile penetration into concrete element of infinite
thickness (inches)

> Note: Usually this equation expresses the depth of pene-
> tration in feet; however, for this document it has been
> modified to express it in inches.

K_p = Penetration coefficient for reinforced concrete (see Figure 2-1).

$A_p = \dfrac{W}{A} = \dfrac{\text{Missile weight}}{\text{Projected frontal area of missile}}$ (psf) | 2

V_s = Striking velocity of missile (ft/sec). (Limit $V_s \leq$ 1000 ft/sec)

This formula is known as the Modified Perry formula.

When the element has a finite thickness the depth of penetration is:

$$X_1 = \left[1 + e^{-4\left(\frac{t}{X} - 2\right)}\right] X, \qquad (t > 2X) \qquad (2\text{-}2) \qquad | 2$$

where

X_1 = Depth of penetration of missile into a concrete element of finite thickness (inches).

e = Base of Napierian Logarithms

t = Thickness of concrete element (inches)

Penetrations for various illustrative examples of missiles are shown in figures 2-2 and 2-4.

2.1.2 PERFORATION

The thickness of a concrete element that will just be perforated by a missile is given by:

$$T = \frac{427}{\sqrt{f'_c}} \frac{W}{D^{1.8}} \left(\frac{V_s}{1000}\right)^{1.33} \tag{2-3}$$

where

T = Thickness of concrete element to be just perforated (inches)

W = Weight of missiles (lb)

D = Diameter of missiles (inches)

> Note: For irregularly shaped missiles, an equivalent diameter is used. The equivalent diameter is taken as the diameter of a circle with an area equal to the circumscribed contact, or projected frontal area, of the non-cylindrical missile.

V_s = Striking velocity of missile (ft/sec)

f'_c = Compressive strength of concrete (psi)

This formula is known as the Ballistic Research Laboratory, BRL, formula.

The thickness, t_p, of a concrete element required to prevent perforation must be greater than T. It is recommended to increase T by 25 percent, but not more than 10 inches, to obtain the t_p, required to prevent perforation

$$t_p = 1.25T \leq T + 10 \text{ (in inches)} \tag{2-4}$$

The threshold of perforation, T, for various illustrative examples of
missiles is shown in figures 2-3 and 2-4.

2.1.3   SPALLING

Spalling of concrete from the side opposite the contact surface of the ele-
ment may occur even if the missile will not perforate the element. For an
estimate of the thickness that will just start spalling, it is recommended
that the following equation be used:

$$T_s = 2T \tag{2-5}$$

where

   T_s = Concrete element thickness that will just start spalling (inches)

   T = Concrete thickness to be just perforated (inches).
       See Equation (2-3)

The thickness, t_s, of a concrete element required to prevent spalling must
be greater than T_s. It is recommended to increase T_s by 25 percent, but
not more than 10 inches, to prevent spalling.

$$t_s = 1.25 \ T_s \leq T_s + 10 \, (\text{in inches}) \tag{2-6}$$

2.2   STEEL TARGETS

Steel targets, such as pipes and mechanical equipment vessels, may be per-
forated by a missile. Sometimes, protruding elements of a missile may
puncture a steel target when the entire missile does not perforate or pass
through the target. The minimum contact area of a missile protrusion is
used to calculate puncture thickness and the projected area of the entire
missile is used to calculate perforation thickness.

The BRL Formula is shown below, modified by setting a material constant
K = 1 and solving directly for steel plate thickness, T, which will just
be perforated by the missile,

$$T = \frac{\left(\dfrac{MV_s^2}{2}\right)^{2/3}}{672D} \tag{2-7}$$

where

T = Steel plate thickness to just perforate (inches).

M = Mass of the Missile (lb sec^2/ft)

W = Weight of the Missile (lb)

V_s = Striking Velocity of the Missile Normal to Target Surface (ft/sec)

D = Diameter of the Missile (in.)

Note: For irregularly shaped missiles the equivalent diameter is used. The equivalent diameter is taken as the diameter of a circle with an area equal to the circumscribed contact, or projected frontal area of the non-cylindrical missile.

The thickness, t_p, of a steel barrier required to prevent perforation should exceed the thickness for threshold of perforations. It is recommended to increase the thickness, T, by 25 percent to prevent perforation.

$$t_p = 1.25T \tag{2-8}$$

2.3 MULTIPLE ELEMENT BARRIERS

It may be desirable to construct a missile barrier of several thinner elements, instead of one thick element. Analysis of missile barriers composed of several elements involves determining the residual velocity (V_r) after perforation of one element and using this value for the striking velocity (V_s) on the next element. The following formula is used to determine the residual velocity, V_r (see Appendix C)

$$
\begin{aligned}
V_r &= \left(V_s^2 - V_p^2 \right)^{1/2} & \text{For } (V_p \leq V_s) \\
V_r &= 0 & \text{For } (V_p \geq V_s)
\end{aligned}
\tag{2-9}
$$

where

V_r = residual velocity of missile after perforation of an element of thickness t. (fps)

V_s = striking velocity of the missile normal to target surface (fps)

V_p = velocity required to just perforate an element (fps)

2.3.1 REINFORCED CONCRETE BARRIER

Combining equations (2-3) and (2-9), the residual velocity of a missile perforating a concrete target is

$$V_r = \left[V_s^2 - \left(\frac{\sqrt{f'_c}\ tD^{1.8}}{427W} \right)^{1.5} 10^6 \right]^{1/2} \tag{2-10}$$

where t = thickness of concrete element (inches)

2.3.2 STEEL BARRIER

Combining equations (2-7) and (2-9), the residual velocity of a missile perforating a steel target is

$$V_r = \left[V_s^2 - \frac{1.12 \times 10^6 (Dt)^{1.5}}{W} \right]^{1/2} \tag{2-11}$$

where t = thickness of steel element (inches)
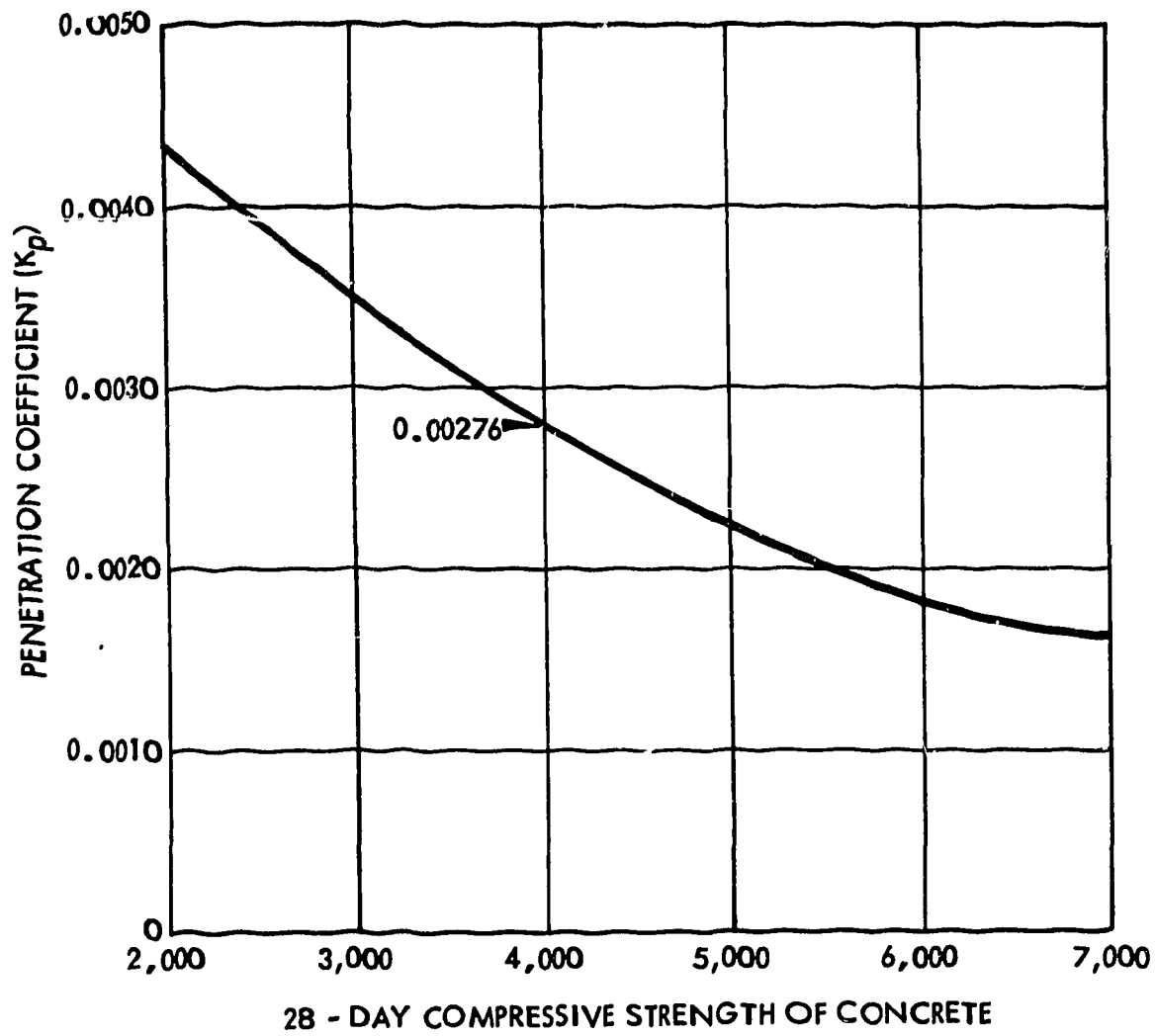
Figure 2-1

VALUES OF PENETRATION COEFFICIENT (K_p) FOR REINFORCED CONCRETE
(Reference 14)
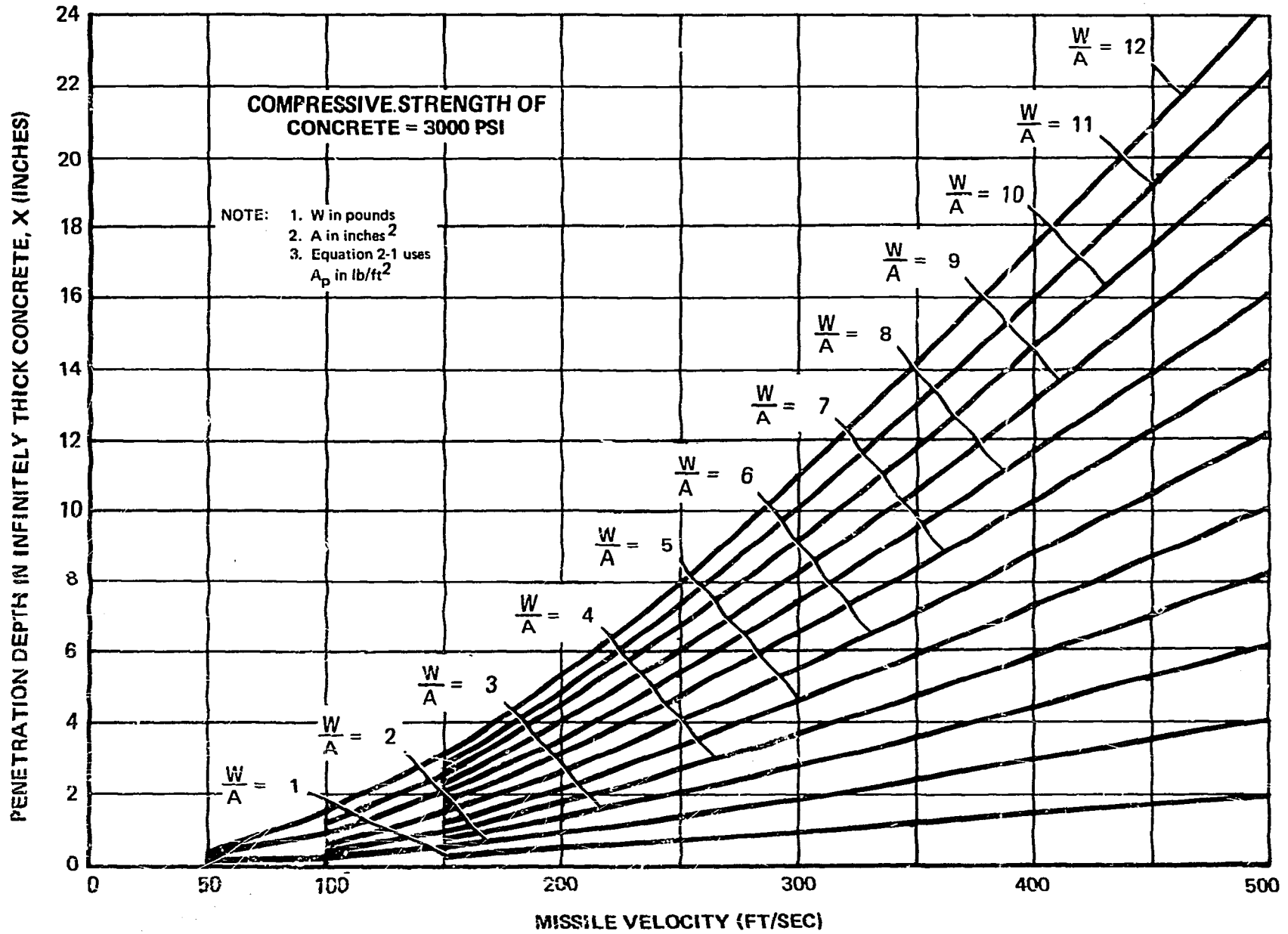
Figure 2-2

PENETRATION OF REINFORCED CONCRETE FOR
VARIOUS MISSILES (MODIFIED PETRY)

CONCRETE THICKNESS TO BE JUST PERFORATED, T (INCHES)

MISSILE VELOCITY (FT/SEC)

COMPRESSIVE STRENGTH OF
CONCRETE = 3000 PSI

NOTE: W in pounds
D in inches

$\dfrac{W}{D^{1.8}} = 12$

$\dfrac{W}{D^{1.8}} = 11$

$\dfrac{W}{D^{1.8}} = 10$

$\dfrac{W}{D^{1.8}} = 9$

$\dfrac{W}{D^{1.8}} = 8$

$\dfrac{W}{D^{1.8}} = 7$

$\dfrac{W}{D^{1.8}} = 6$

$\dfrac{W}{D^{1.8}} = 5$

$\dfrac{W}{D^{1.8}} = 4$

$\dfrac{W}{D^{1.8}} = 3$

$\dfrac{W}{D^{1.8}} = 2$

$\dfrac{W}{D^{1.8}} = 1$
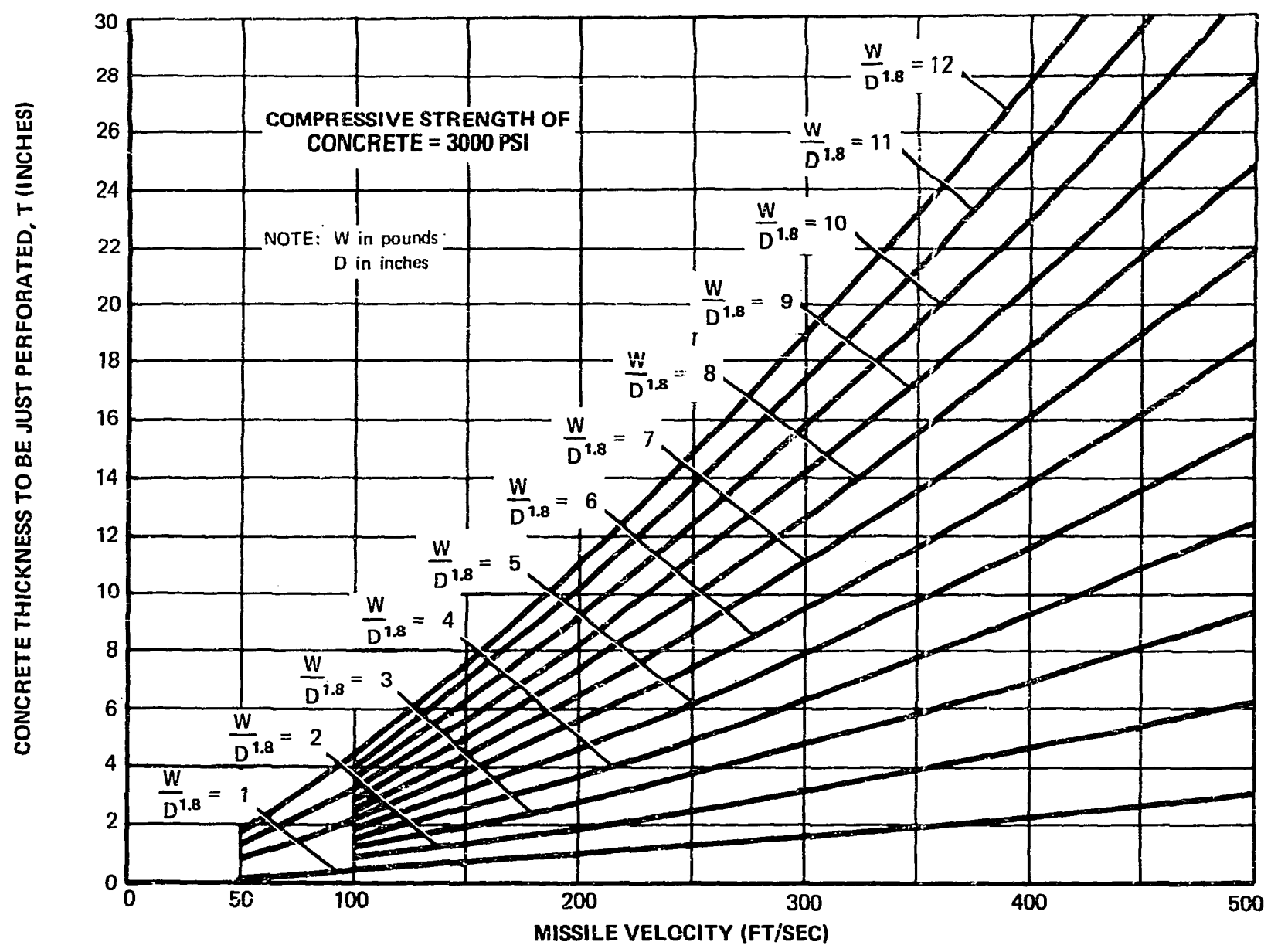
Figure 2-3

PERFORATION OF REINFORCED CONCRETE FOR
VARIOUS MISSILES (BALLISTIC RESEARCH LAB)

PENETRATION DEPTH
IN INFINITE THICKNESS CONCRETE, EQ. 2-1 (INCHES)

LEGEND:

① 4X12 WOOD PLANK 200 LBS.   ④ 6" φ STEEL PIPE 285 LBS.
② 1" φ STEEL ROD 8 LBS.   ⑤ 12" φ STEEL PIPE 743 LBS.
③ 3" φ STEEL PIPE 78 LBS.   ⑥ 13.5" φ UTILITY POLE 1490 LBS.

NOTE:
COMPRESSIVE STRENGTH
OF CONCRETE = 4000 PSI

MISSILE VELOCITY (FT/SEC)

CONCRETE THICKNESS FOR THRESHOLD OF PERFORATION, EQ. 2-3

CONCRETE THICKNESS FOR THRESHOLD OF SPALLING, EQ. 2-5 (INCHES)

Figure 2-4

PENETRATION, PERFORATION, AND SPALLING OF REINFORCED
CONCRETE TARGETS BY POSTULATED TORNADO MISSILES

Section 3

STRUCTURAL RESPONSE TO MISSILE IMPACT LOAD

3.1 GENERAL

When a missile strikes a target, large forces develop at the missile -
target interface, which decelerate the missile and accelerate the target.
If the interface forcing function is known, (experimentally determined),
the target structure can be modeled mathematically and conventional numeri-
cal techniques can be used to predict structural response. For most cases,
the forcing function is not known, and a rational method involving energy
balance techniques is used to estimate structural response. This involves
using the strain energy of the target at maximum response to balance the
residual kinetic energy of the target (or target-missile combination)
resulting from missile impact.

For investigation purposes, it is convenient to model the event as a missile
of mass, M_m, and striking velocity, V_s, impacting a spring-backed target
mass, M_e. The spring may be linear, bilinear, or non-linear, depending on
the target structure resistance-displacement function. Since the actual
coupled mass varies during impact, an estimated average effective target
mass, M_e is used to evaluate inertia effects during impact.

The impact may be either elastic or plastic, depending on whether or not
significant energy losses are sustained during impact. These losses are
associated with inelastic deformations, local damage in the impact zone,
etc.

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Plastic impact is characterized by the missile remaining in contact with
the target, subsequent to impact. In an elastic impact, the missile and
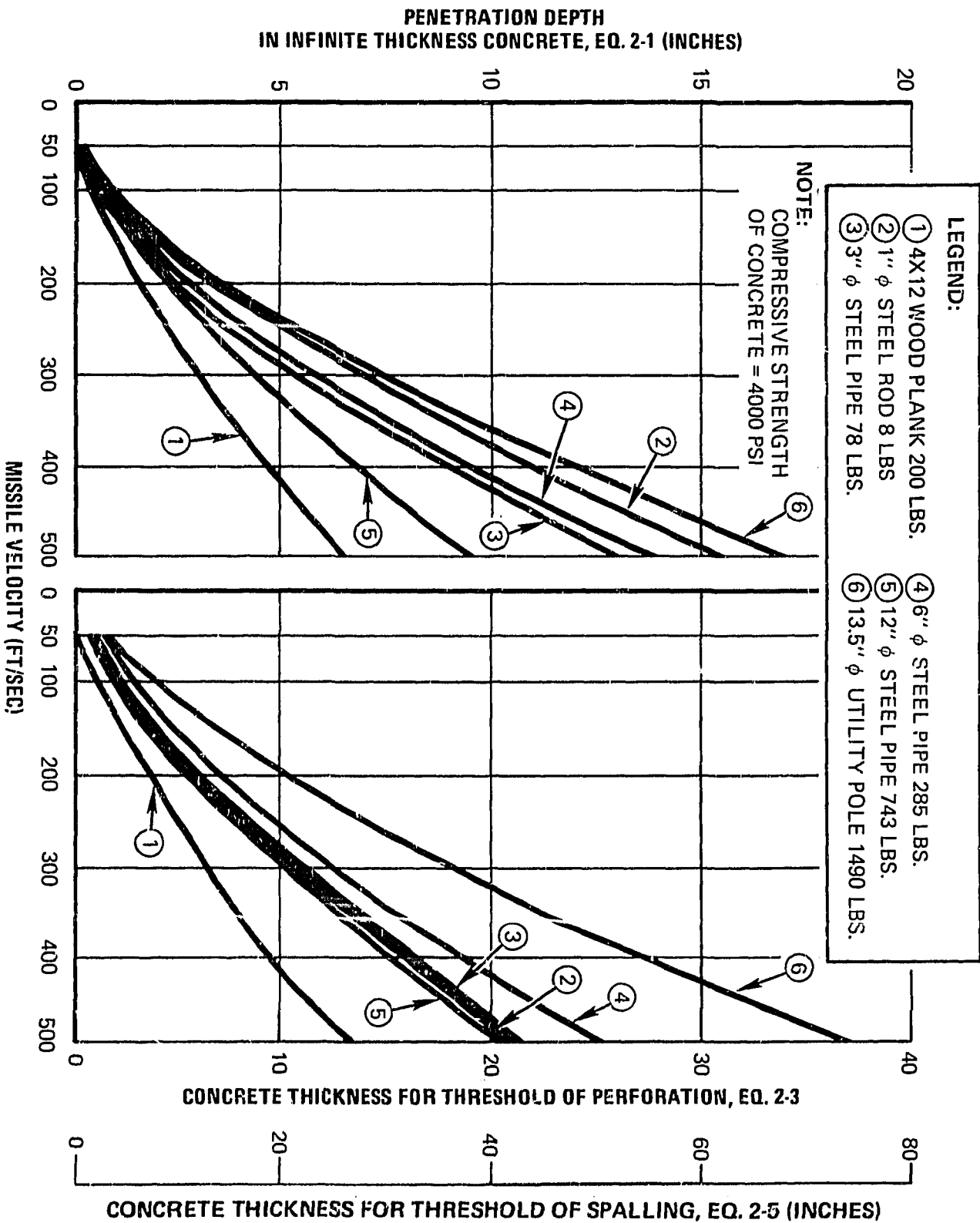target remain in contact for a very short period of time, and then disen--
gage due to elastic interface restoring forces.

An elastic missile impact case is rarely encountered in nuclear plant
design. For example, based on information available, a plastic collision
can be considered for all postulated tornado-generated missiles.

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3.2 VELOCITY AFTER IMPACT

Since the duration of impact is very short, (usually less than a few milli-
seconds), the target mass displacement and the corresponding spring force
are also very small. Neglecting the spring force effect during impact,
(a slight conservatism), the velocities of the missile and target after
impact are calculated from the following relationships:

$$V_m = \frac{V_s (M_m - eM_e)}{M_m + M_e} \qquad (3-1)$$

$$V_T = \frac{V_s M_m (1+e)}{M_m + M_e} \qquad (3-2)$$

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V_m = Missile velocity after impact

V_T = Target velocity after impact

V_o = Missile striking velocity

M_m = Mass of missile

M_e = Effective mass of target during impact

e = Coefficient of restitution

3.3   REQUIRED TARGET STRAIN ENERGY CAPACITY

3.3.1   ELASTIC IMPACT

Equations (3-1) and (3-2)[12]* show that the velocity of the missile after impact is opposite to that of the target if M_m is less than eM_e. For this case, the strain energy, E_S, of the responding target spring required to diminish the target mass velocity to zero (maximum target response) is numerically equal to the kinetic energy of the target mass at the end of the impact duration.

$$E_s = \frac{M_e V_T^{\,2}}{2} \qquad (3-3)$$

If the impact is determined to be elastic and the coefficient of restitution is not known, a conservative value of e equal to unity can be assumed. Making this substitution in equation (3-2), and substituting this value for V_T into equation 3-3, the required strain energy of the responding target is;

$$E_s = \frac{2M_m^2 M_e V_s^2}{\left(M_m + M_e\right)^2} \qquad (3-4)$$

Referring again to equations (3-1) and (3-2), the velocity of the missile after impact is in the same direction as that of the target if M_m is greater than eM_e. In this case, the target spring decelerates the target mass, allowing the missile to overtake the target, which results in multiple impact.

If the impact is purely elastic (e = 1), the target will eventually stop the missile through a series of impacts and absorb all the initial kinetic

*References are in appendix F.

energy of the missile. The required strain energy of the responding target is then equal to the initial kinetic energy of the missile.

$$E_s = \frac{M_m V_s^2}{2} \qquad (3-5)$$

3.3.2   PLASTIC IMPACT

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For a plastic collision, the coefficient of restitution reduces to zero ($e = 0$) and the missile and target masses attain the same velocity at the end of impact duration. If the impact is of short duration, the target displacement and corresponding spring force effect during impact are small, and can be conservatively neglected. The strain energy required to stop the target-missile combination is then the sum of the kinetic energy of the missile and the target masses at the end of the duration of impact.

$$E_s = \frac{M_m V_m^2}{2} + \frac{M_e V_T^2}{2} \qquad (3-6)$$

From equations 3-1 and 3-2

$$V_m = V_T = \frac{M_m V_s}{M_m + M_e} \qquad (3-7)$$

Substituting the value for V_m and V_T from equation (3-7) into equation (3-6), the required target strain energy is

$$E_s = \frac{M_m^2 V_s^2}{2 (M_m + M_e)} \qquad (3-8)$$

3.3.3   FORCE TIME FUNCTION KNOWN

In some isolated cases, (such as for frontal impact of an automobile, see section 5.1), sufficient experimental data are available to enable definition of a force-time function, F(t), at the interface between the missile and target. This enables direct solution of the equation of motion:

$$F(t) - R(x) = M_e \ddot{x} \qquad (3-9)$$

F(t) = Force-time function

R(x) = Resisting spring force as a function of displacement, x

$\ddot{x}$ = Acceleration of target mass

M_e = Effective target mass

Numerical methods are usually used for solution of equation 3-9 which is solved for the maximum value of displacement x_m. The target strain energy is then;

$$E_s = \int_0^{x_m} R(x)dx$$

An abbreviated conservative solution for required target strain energy can be obtained if $R(x)$ during impact is small compared to $F(t)$ and plastic or permanent deformation is dominant at the missile-target interface

The velocity of the target mass at time, t, is;

$$\dot{x}(t) = \int_0^t \ddot{x}dt = \int_0^t \frac{[F(t) - R(x)] \ dt}{M_e}$$

The kinetic energy of the target mass at time t is then

$$E(t) = \frac{M_e[\dot{x}(t)]^2}{2}$$

or

$$E(t) = \frac{1}{2M_e} \left\{ \int_0^t [F(t) - R(x)] \ dt \right\}^2 \tag{3-10}$$

Equation (3-10) shows that deletion of the $R(x)$ term will result in a conservative overestimate of $E(t)$. If $R(x) << F(t)$ during impact, t, the inaccuracy is usually negligible. For this condition, the kinetic energy of the target mass at time t_i is conservatively estimated as;

$$E_{t_i} = \frac{\left[\int_0^{t_i} F(t) \ dt \right]^2}{2M_e} \tag{3-11}$$

The applied impulse, I, is by definition, the area under the force-time curve.

$$I = \int_0^{t_i} F(t) \ dt$$

Making this substitution into equation (3-11);

$$E_{t_i} = \frac{I^2}{2M_e} = (1/2)M_e V_T^2 \qquad (3-12)$$

If the elastic restoring forces at the missile-target interface are small, the velocity of the missile approaches that of the target at the end of time, t_i, equal to the duration of impact. The strain energy of the target required to stop the missile-target combination is then;

$$E_s = \frac{M_m V_m^2}{2} + \frac{I^2}{2M_e} \qquad (3-13)$$

For a plastic collision,

$$V_m = V_T$$

From equation (3-12):

$$V_T^2 = \frac{I^2}{M_e^2}$$

and,

$$V_m^2 = \frac{I^2}{M_e^2}$$

Making this substitution into equation (3-13):

$$E_s = \frac{(M_m + M_e) I^2}{2M_e^2} \qquad (3-14)$$

3.4 TARGET EFFECTIVE MASS

The effective target mass during impact varies from a low value at initial contact and generally increases to an upper limit during or at the end of the impact duration. Due to the complex phenomenology associated with missile impact, no general analytical solution is available to evaluate the effective coupled mass on a continuous time basis. The average effective mass can, however, be estimated, utilizing the results of impact tests on reinforced concrete beams[7] wherein the measured maximum structural response was used to back-calculate the average mass during impact.

Based on these data, the following formulae provide a lower limit estimate of M_c (which results in an upper limit estimate of kinetic energy after impact).

For concrete beams:

2

$$M_e = (D_x + 2T) \frac{B\gamma_c T}{g}, \qquad \left[\text{if } B \leq (D_y + 2T) \right]$$

$$M_e = (D_x + 2T)(D_y + 2T) \frac{\gamma_c T}{g}, \qquad \left[\text{if } B \geq (D_y + 2T) \right] \qquad (3-15)$$

For concrete slabs:

$$M_e = (D_x + T)(D_y + T) \frac{\gamma_c T}{g} \qquad (3-16)$$

For steel beams:

$$M_e = (D_x + 2d) M_x \qquad (3-17)$$

For steel plates

$$M_e = D_x D_y \frac{\gamma_s t}{g} \qquad (3-18)$$

M_e = Average effective mass of target during impact

M_x = Mass per unit length of steel beam

D_x = Maximum missile contact dimension in the x direction (longitudinal axis for beams or slabs)

D_y = Maximum missile contact dimension in the y direction (transverse to longitudinal axis for beams or slabs)

T = Thickness or depth of concrete element

t = Thickness of steel plate

d = Depth of steel beam

B = Width of concrete beam (not to exceed $D_y + 2T$)

γ_c = Weight per unit volume of concrete

γ_s = Weight per unit volume of steel

g = Acceleration of gravity

3.5   STRUCTURAL RESPONSE BY ENERGY BALANCE METHOD

3.5.1   GENERAL PROCEDURES

The strain energy, E_s, required to stop the target (or missile-target combination), is determined from the relationships in sections 3.2 and 3.3.

The resistance-displacement function, $R(x)$, for a concentrated load at the area of impact is determined from the target structure physical configuration and material properties.

The estimated maximum target response is determined by equating the available target strain energy to the required strain energy and solving for the maximum displacement x_m. (See Figure 3-1.)

3.5.2   ELASTIC TARGET RESPONSE

For elastic response,

$R(x) = kx$

k = Elastic spring constant

If no other loads are acting concurrently with the missile impact loading, the maximum response is

$$x_m = \left[\frac{2\ E_s}{k}\right]^{1/2} \tag{3-19}$$

If other loads are present on the target structure which will act concurrently with missile impact loads, the maximum combined displacement is determined as follows:

Then

$$x_m = x_o + x'$$

Since

$$x' = \left[\frac{2\ E_s}{k}\right]^{1/2}$$

Let

x' = Displacement due to missile impact (See Figure 3-1)

x_o = Displacement due to other loads

x_m = Maximum combined displacement

2| it follows that

$$x_m = x_o + \left[\frac{2\ E_s}{k}\right]^{1/2} \qquad (3-20)$$

3.5.3 ELASTO-PLASTIC TARGET RESPONSE

For elasto-plastic target response with no other concurrent loads acting:

$R(x) = kx, \quad \left(0 < x \leq x_e\right)$

$R(x) = k x_e = R_m, \quad (x_e < x \leq x_m)$

where

x_e = Yield displacement

R_m = Plastic resistance.

Then

$E_s = R_m \left(x_m - \frac{x_e}{2}\right)$

or

$$x_m = \frac{E_s}{R_m} + \frac{x_e}{2} \qquad (3-21)$$

The required ductility ratio, μ_r, is obtained from equation (3-21) by dividing both sides of the equation by x_e.

$$\mu_r = \frac{x_m}{x_e}$$

$$\mu_r = \frac{E_s}{x_e R_m} + \frac{1}{2} \qquad (3-22)$$

If other loads are present on the target structure which will act concurrent with missile impact loads, the maximum combined displacement is determined as follows:

Let

$x' = x_e - x_o$ (see figure 3-1)

x_o = displacement due to other loads

x_e = yield displacement

x_m = maximum combined displacement

R_m = plastic resisting force

k = elastic spring constant

Then

$$E_s = \frac{k(x')^2}{2} + kx' \, (x_m - x_e) \quad \text{(see figure 3-1)}$$

or

$$x_m = \frac{E_s}{kx'} - \frac{x'}{2} + x_e$$

Substituting $x' = x_e - x_o$ in the above equation gives

$$x_m = \frac{E_s}{k \, (x_e - x_o)} + \frac{x_e + x_o}{2} \tag{3-23}$$

The required ductility ratio, μ_r, is obtained by dividing both sides of equation (3-23) by x_e.

$$\mu_r = \frac{E_s}{R_m \, (x_e - x_o)} + \frac{1 + x_o/x_e}{2} \tag{3-24}$$

The values of μ_r should be less than the allowable ductility ratios μ given in section 4.

3.5.4   NON-LINEAR TARGET RESPONSES

If the resistance-displacement function is nonlinear (figure 3-1) the determination of structural response is facilitated by first defining the strain energy-displacement function, (see figure 3-2).

$$E_e = \int_0^x R(x) \, dx \tag{3-25}$$

E_e = strain energy at displacement x

E_e = strain energy at displacement x

When no other concurrent loads are acting, the maximum displacement occurs at the value of x where E_e is equal to E_s. The correct value of x_m is therefore the value of x, which will satisfy the following relationship:

$$E_s = \int_0^{x_m} R(x) \, dx \qquad (3\text{-}26)$$

A typical graphical solution is shown in figure 3-2.

When other loads are acting concurrent with missile impact loading, the correct value of x_m will satisfy the following relationship:

$$E_s = \int_{x_0}^{x_m} R(x)dx - R_0(x_m - x_0) \qquad (3\text{-}27)$$

R_0 = equivalent static resistance required for other loads (see figure 3-1)

x_0 = displacement associated with R_0.

A typical graphical solution for x_m is shown schematically in figure 3-3.

To provide an adequate margin of safety the values of E_s should satisfy the condition

$$E_s \leq F_s E_f \qquad (3\text{-}28)$$

E_f = impact strain energy capacity

F_s = safety factor

F_s = 0.5 if R(x) is well defined from tests

F_s = 0.25 if R(x) is approximately determined (such as by failure analysis)

For impact only:

$$E_f = \int_0^{x_f} R(x) \, dx \qquad (3\text{-}29)$$

x_f = displacement at failure

For impact combined with other loads:

$$E_f = \int_{x_o}^{x_f} R(x)\,dx - R_o\,(x_f - x_o) \qquad (3-30)$$

and

$$R_o \leq R_f$$

R_f = resistance at failure

RESPONSE	RESISTANCE – DISPLACEMENT FUNCTION	AVAILABLE STRAIN ENERGY WITHOUT OTHER LOADING	AVAILABLE STRAIN ENERGY WITH OTHER LOADING
ELASTIC	$R = kx$	x_m	x_o x_m
ELASTO-PLASTIC	$R = kx$ R_m x_e	x_e x_m	x_o x_e x_m
NON-LINEAR	$R = R(x)$	x_m x_f	x_o x_m x_f R_o
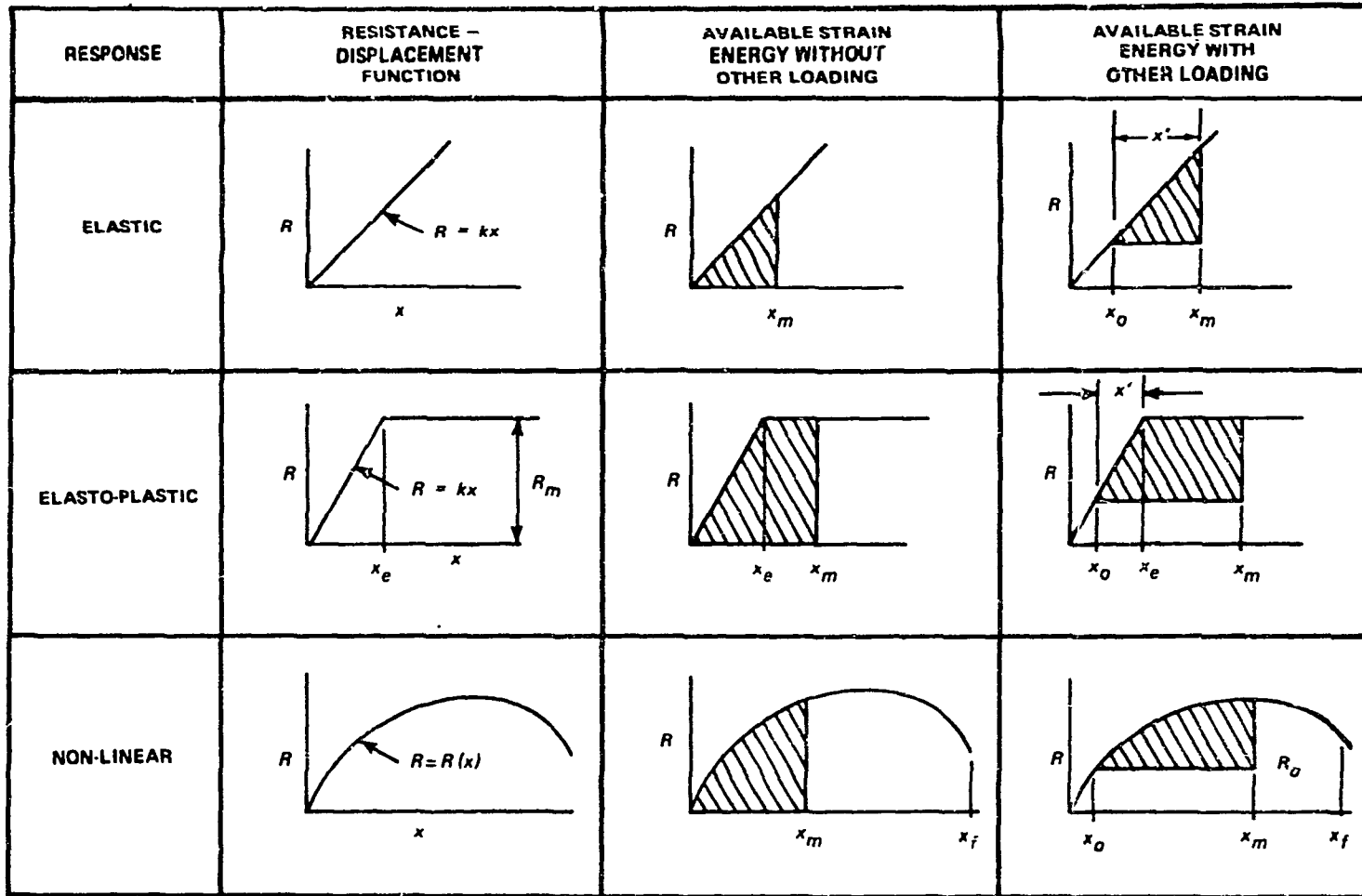
SHADED AREA (STRAIN ENERGY) MUST EQUAL E_s (FROM SECTIONS 3.2 AND 3.3)

Figure 3-1

RESISTANCE-DISPLACEMENT FUNCTIONS WITH ASSOCIATED STRUCTURAL
RESPONSE WITH AND WITHOUT THE EFFECT OF OTHER LOADS

$$E_a = \int_0^x R(x)\,dx$$

ENERGY E_a

E_s

E_f

x_m x_f

DISPLACEMENT X

Figure 3-2

ENERGY–DISPLACEMENT FUNCTIONS–
IMPACT LOADS ONLY

3-13

Figure 3-3

ENERGY-DISPLACEMENT FUNCTIONS –
IMPACT COMBINED WITH OTHER LOADS

Section 4

DESIGN GUIDELINES

4.1 ALLOWABLE STRESSES AND LOADINGS

The combination of loadings, allowable stress and strain limits, and
applicable codes used with the missile impact loading are given in the
Safety Analysis Report. The resistance of a structural component must be
based on its minimum strength, i.e., the minimum of its flexural or shear-
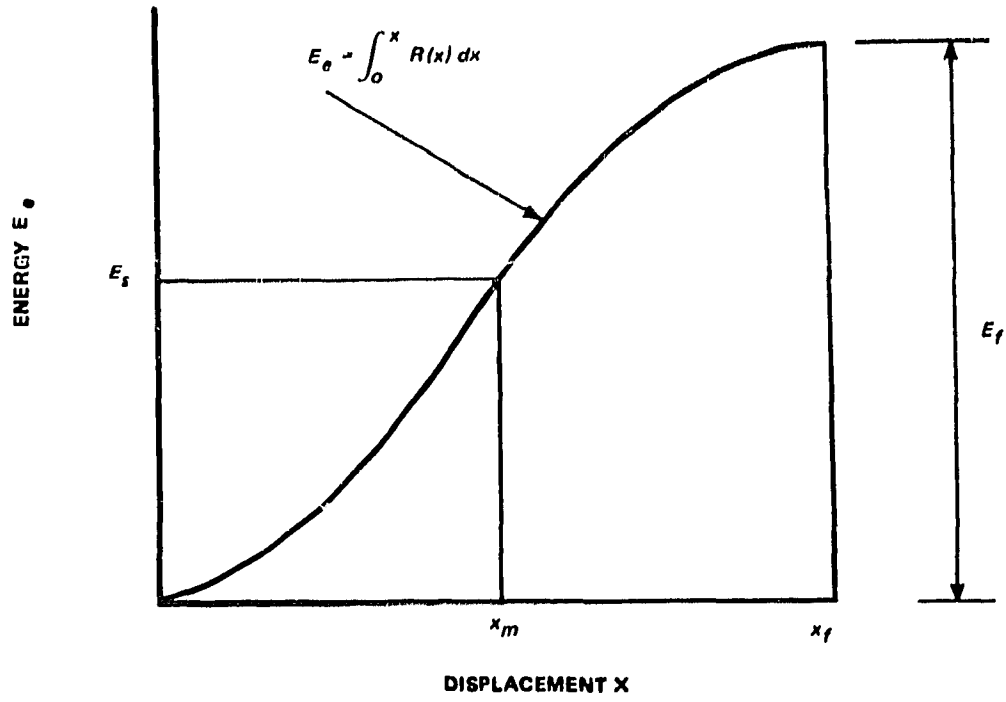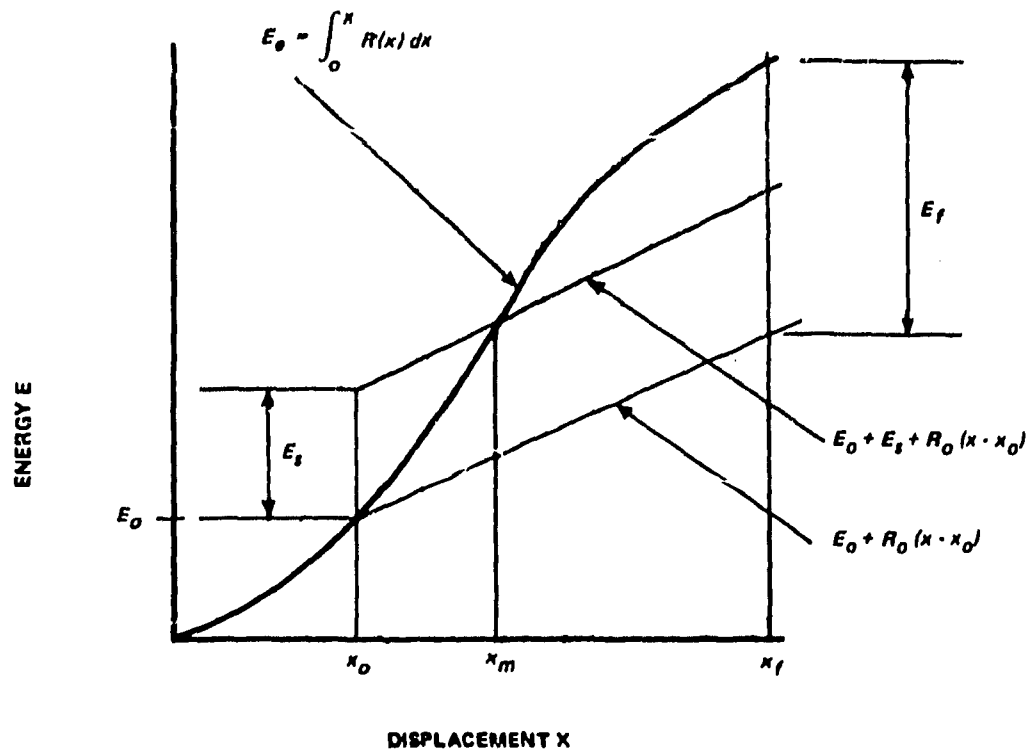ing capacity. The dynamic capacity of the structural elements must be
based on material dynamic strength properties which are obtained by applying
a dynamic increase factor (DIF) to the static strength value:

$$f_{dyn} = (DIF) \ f_{stat} \qquad (4-1)$$

where

f_{dyn} = allowable dynamic strength value

f_{stat} = specified static strength value

DIF = dynamic increase factor

The dynamic increase factor for various materials are given in table 4-1.

4.2 DESIGN PARAMETERS

The resistance of typical structural elements, whose flexural strength
defines the minimum capacity, and their yield displacement approximations
are presented in tables 4-2 and 4-3. Similar equations can be developed
for the load at other location on the structural element. It is prefer-
able that the limiting capacity of an element be in the flexural mode not
in shear. In evaluating the yield displacement with the usual elastic
analysis, the moment of inertia must account for cracking of concrete
sections. The empirical relation for this type of loading is an average
moment of inertia I_a calculated as follows is:

$$I_a = \frac{1}{2} \left(I_g + I_c \right) = \frac{1}{2} \left(\frac{bt^3}{12} + Fbd^3 \right) \qquad (4-2)$$

where

I_g = moment of inertia of gross concrete cross section of thickness t
about its centroid (neglecting steel areas)

I_c = moment of inertia of the cracked concrete section

b = width of concrete section

F = coefficient for moment of inertia of cracked section with tension reinforcing only. (See figure 4-1.)

t = concrete thickness

d = distance from extreme compression fiber to centroid of tension reinforcing

The moment of inertia I_a, as calculated by equation (4-2), must be used in the displacement equation in tables 4-2 and 4-3 for all reinforced concrete members. The ultimate moment capacity of a concrete section shall be considered as the moment strength

$$M_u = 0.9 \, A_s \, f_{dy} \, (d - a/2) \tag{4-3}$$

where

A_s = area of tensile reinforcing steel

f_{dy} = allowable dynamic yield stress for reinforcing steel

d = distance from extreme compression fiber to centroid of tension reinforcing

a = depth of equivalent rectangular stress block

If the element has compression steel, it should be considered and the appropriate equation used.

The amount of reinforcing steel in a concrete members must satisfy the following criteria:

For members with tension steel only:

$$\frac{1.4\sqrt{f'_c}}{f_y} \left(\frac{t}{d}\right)^2 \leq \frac{A_s}{bd} \leq \frac{0.25 \, f'_c}{f_y} \tag{4-4}$$

For members with tension and compression steel:

$$\frac{1.4\sqrt{f'_c}}{f_y} \left(\frac{t}{d}\right)^2 \leq \frac{A_s}{bd} \tag{4-4a}$$

$$\frac{A_s - A'_s}{bd} \left(\frac{t}{d}\right)^2 \leq \frac{0.25 \, f'_c}{f_y}$$

where

f'_c = compression strength of concrete

A'_s = area of compressive reinforcement of concrete

4.3 ALLOWABLE DUCTILITY RATIO

The maximum allowable ductility ratios for concrete and steel members are
presented in Table 4-4. However, the maximum deflection shall be limited
so as not to impair the function of other safety related equipment.

2

Table 4-1

DYNAMIC INCREASE FACTOR
(DIF)
(From Ref. 19)
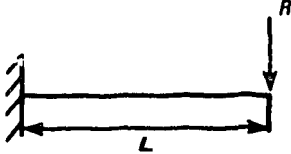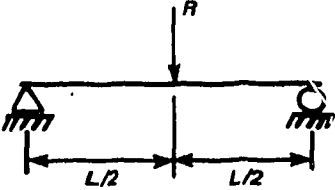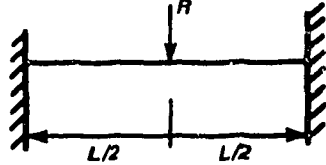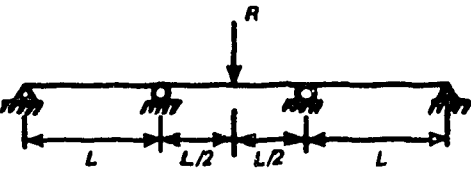
I. Reinforced or Prestressed Concrete

Concrete	DIF
Compression	1.25
Diagonal Tension & Direct Shear (Punch Out)	1.0
Bond	1.0

Reinforcing Steel

Tension & Compression For 40ksi yield strength steel		1.2
	60ksi yield strength steel	1.0
Diagonal Tension & Direct Shear (Stirrups)		1.0

2

II. Structural Steel

Flexure, Tension, & Compression for 40ksi yield strength steel		1.2
	60ksi yield strength steel	1.0
Shear		1.0

2

Table 4-2

RESISTANCE-YIELD DISPLACEMENT
VALUES FOR BEAMS

DESCRIPTION	RESISTANCE	YIELD DISPLACEMENT

(1) CANTILEVER

$$R = \frac{M_u}{L}$$

$$x_e = \frac{RL^3}{3EI}$$

$\Big|$ 2

(2) SIMPLY SUPPORTED

$$R = \frac{4M_u}{L}$$

$$x_e = \frac{RL^3}{48EI}$$

$\Big|$ 2

(3) FIXED SUPPORTS

$$R = \frac{4(M_u^+ + M_u^-)}{L}$$

$$x_e = \frac{RL^3}{192EI}$$

(4) MULTI-SPAN

$$R = \frac{4(M_u^+ + M_u^-)}{L} \qquad x_e = \frac{0.011RL^3}{EI}$$

2

Where M_u^+ = ULTIMATE POSITIVE MOMENT CAPACITY

M_u^- = ULTIMATE NEGATIVE MOMENT CAPACITY

I = MOMENT OF INERTIA (in^4)

FOR REINFORCED CONCRETE $I = I_s$, SEE EQUATION 4-2.

Table 4-3

RESISTANCE YIELD DISPLACEMENT
VALUES FOR SLABS
AND PLATES
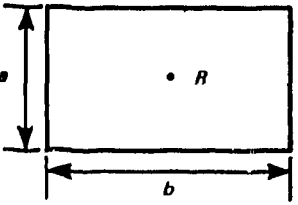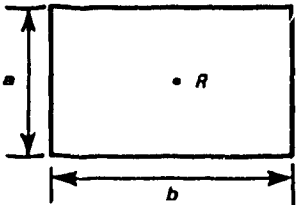
	DESCRIPTION	RESISTANCE (29, 30, 31, 32)	YIELD DISPLACEMENT (33)

2

(1) SIMPLY SUPPORTED ON ALL 4 SIDES WITH LOAD AT CENTER

2

$R = 2\pi M_u$

$X_e = \dfrac{\alpha R a^2}{12EI} \ (1-\nu^2)$

b/a	1.0	1.1	1.2	1.4	1.6	1.8	2.0	3.0	∞
α	.1390	.1518	.1624	.1781	.1884	.1944	.1981	.2029	.2031

(2) FIXED SUPPORTS ON ALL 4 SIDES WITH LOAD AT CENTER

2

ν = POISSON'S RATIO

t = THICKNESS (in)

E = MODULUS OF ELASTICITY (lb/in^2)

I = MOMENT OF INERTIA PER UNIT WIDTH (in^4/in) FOR REINFORCED CONCRETE SECTION I = I$_a$, SEE EQUATION 4-2

M_u^+ = ULTIMATE POSITIVE MOMENT CAPACITY (in lb/in)

M_u^- = ULTIMATE NEGATIVE MOMENT CAPACITY (in lb/in)

$R = 2\pi (M_u^+ + M_u^-)$

$X_e = \dfrac{\alpha R a^2}{12EI} \ (1-\nu^2)$

b/a	1.0	1.2	1.4	1.6	1.8	2.0	∞
α	.0671	.0776	.0830	.0854	.0864	.0866	0.0871

2

Table 4-4

DUCTILITY RATIOS
(From Reference 28)

Max. Allowable Value of μ | 2

Reinforced Concrete

Flexure

Beams	$\dfrac{0.10}{p-p'} \leq 10$
Slabs	$\dfrac{0.10}{p-p'} \leq 30$

Compression

Walls & Columns	1.3

where

p is the ratio of tensile reinforcement $= \dfrac{A_s}{bd}$

p' is the ratio of compressive reinforcement $= \dfrac{A'_s}{bd'}$

Steel Elements

Members proportioned to preclude lateral
and local buckling

Flexure, compression and shear	20

Steel Columns | 2

Proportioned to preclude elastic buckling	1.3
Members stressed in tension only	$0.5\,\dfrac{e_u}{e_y}$

e_u = ultimate strain

e_y = yield strain

2

$$I_{cr} = F\ bd^3$$

$$\left(\frac{p'}{p}\right)\begin{cases}1.0\\0.75\\0.50\\0.25\\0.00\end{cases}$$

COEFFICIENT F

RATIO pn
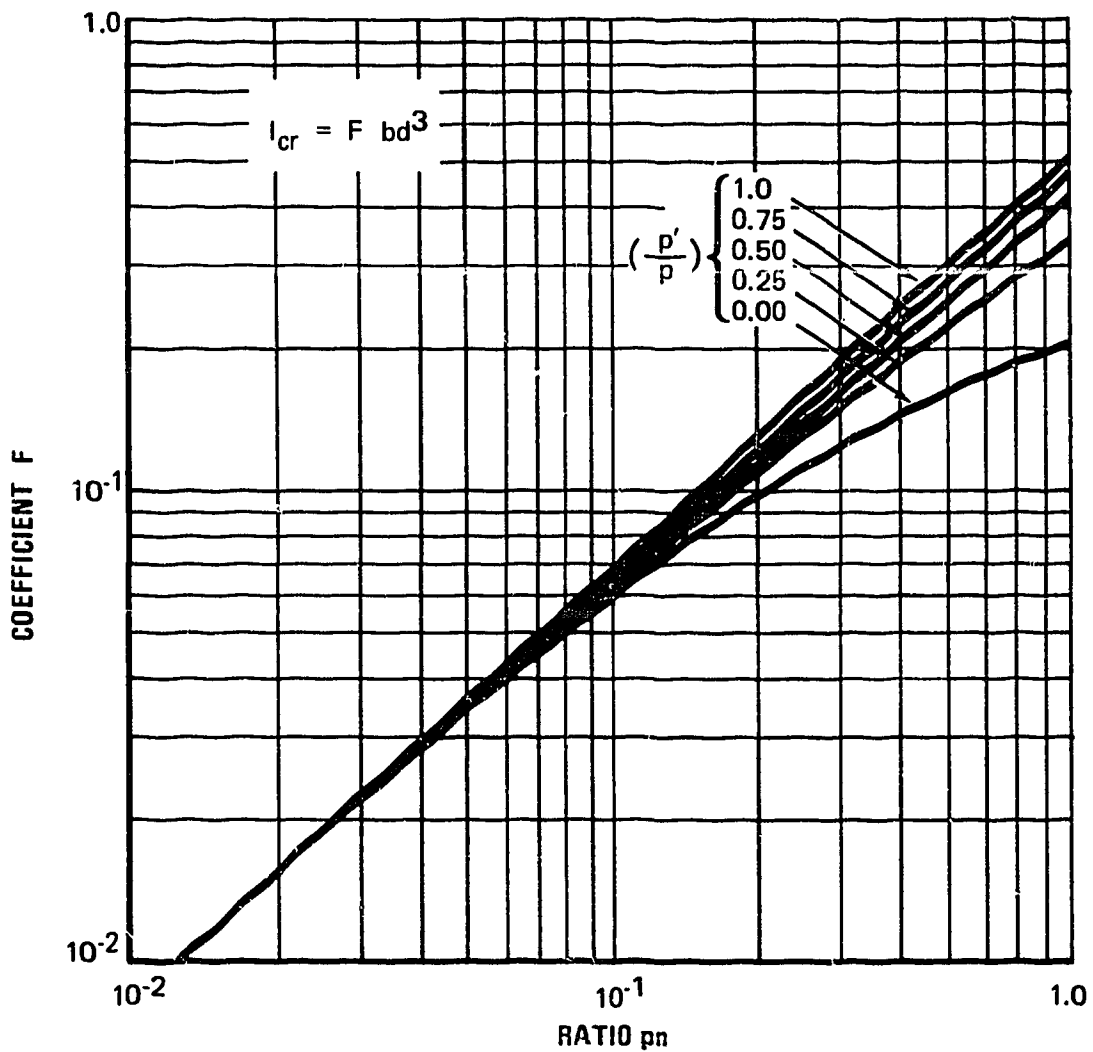
$$p = \frac{As}{bd}, \qquad p' = \frac{A's}{bd}, \qquad n = \frac{Es}{Ec}$$

$$F = \frac{K^3}{3} + pn\,(1\text{-}K)^2 \quad + \left(\frac{2n\text{-}1}{n}\right)\ (pn)\frac{p'}{p}\ (K - \frac{d'}{d})^2$$

$$\frac{2n\text{-}1}{n} \cong 1.9, \qquad \frac{d'}{d} \cong 0.10, \qquad K = -m + (m^2 + 2q)^{\frac{1}{2}}$$

$$m = pn\,(1 + 1.9\frac{p'}{p}), \qquad q = pn\,(1 + 0.19\frac{p'}{p})$$

Figure 4-1

COEFFICIENTS FOR MOMENT OF INERTIA
OF CRACKED SECTIONS

SECTION 5

SPECIAL PROBLEMS

Two special problems are the determination of an empirical formula for force-time history of automobile crash, and the evaluation of a missile's velocity as it passes through a liquid.

5.1 FORCE-TIME HISTORY FOR AUTOMOBILE CRASH

In deriving the force-time history of an automobile crash under frontal impact, the automobile is considered as a deformable missile and the structure as a rigid target. According to Appendix D, Paragraph D.1, which is based on a theoretical consideration and considerable experimental data, the force-time history under such a condition is approximately as follows:

$$F(t) = 0.625 \ V_s \ W_m \ \sin 20t, \qquad (0 \leq t \leq 0.0785 \ sec)$$

$$F(t) = 0 \qquad\qquad\qquad (t > 0.0785 \ sec)$$

(5-1)

where

t = time from the instant of initial contact (sec)

$F(t)$ = time-dependent force on target (lb)

V_s = striking velocity of the automobile (ft/sec)

W_m = weight of automobile (lb)

References on derivations of more elaborate force-time histories for automobile crashes are given in reference 11.

5.2 PENETRATION OF A MISSILE THROUGH A LIQUID

To evaluate the effect of a missile on a target that is submerged in a liquid, determine the striking velocity of the missile, V, after it has penetrated through a depth, H, of liquid covering the target (figure 5-1). This involves evaluating the velocity change due to missile weight, the buoyant force, and the drag force.

The penetration of a missile as it enters a liquid depends on the geometric shape of the missile. For the vertical entry of a missile with uniform horizontal cross-sectional area A_0, and length L, the depth of penetration and the velocity at a depth, x, are in terms of two functions of x. (The functions are evaluated at $x = H$ or L.)

$$Z_1(x) = g/a + bA_0(1-2ax)/2a^2 + e^{-2ax}(V_0^2 - g/a - bA_0/2a^2), \quad (0 \le x \le L) \quad (5-1)$$

$$Z_2(x) = V_2^2 + e^{-2ax}\left\{ bA_0\left[e^{2aL}(1-2aL)-1 \right]/2a^2 + V_0^2 + g(e^{2aL}\ Y/Y_m-1)/a \right\},$$

$$(x \ge L) \quad\quad\quad (5-2)$$

Notations used above are defined at the end of this section. Missile penetration in a liquid can be catagorized by the following cases:

5.2.1   LIQUID DEPTH IS LESS THAN OR EQUAL TO MISSILE LENGTH ($H \le L$)

5.2.1.1   If $Z_1(x)$ is Negative or Zero at Depth $x = H$ ($Z_1(H) \le 0$)

The missile will not strike the target. It will penetrate a depth $H_1 \le H$ such that $Z_1(H_1) = 0$, and then float to the liquid surface.

5.2.1.2   If $Z_1(x)$ is Positive at Depth $x = H$ ($Z_1(H) > 0$)

The striking velocity at depth H is

$$V = \left[Z_1(H) \right]^{1/2} \quad\quad\quad (5-3)$$

5.2.2   LIQUID DEPTH IS GREATER THAN MISSILE LENGTH ($H > L$)

5.2.2.1   If $Z_2(x)$ is Negative or Zero at Depth $x = L$ ($Z_2(L) \le 0$)

The missile will not strike the target. It will penetrate a depth $H_1 \le L$ such that $Z_1(H_1) = 0$, and then float to the liquid surface.

5.2.2.2   If $Z_2(x)$ is Positive at Depth $x = L$ ($Z_2(L) > 0$)

The missile will penetrate the liquid deeper than L. There are two possibilities:

A.   If $Z_2(x)$ is Negative or Zero at Depth $x = H$ ($Z_2(H) \le 0$)

   The missile will not strike the target. It will penetrate a depth H_2 ($L < H_2 \le H$) such that $Z_2(H_2) = 0$, and then float to the liquid surface.

B.   If $Z_2(x)$ is Positive at Depth $x = H$ ($Z_2(H) > 0$)

   The striking velocity at depth H is

$$V = \left[Z_2(H) \right]^{1/2} \quad\quad\quad (5-4)$$

In case the missile shape does not have a uniform cross-sectional area, refer to equations (D-23) and (D-36) in Appendix D.2 for more general solutions.

5.2.3 DEFINITIONS OF NOTATIONS

$$a = \gamma A_0 C_D/2W \qquad (5-5)$$

$$b = \gamma g/W \qquad (5-6)$$

g = gravitational acceleration
($g = 32.17$ ft/sec^2 at sea level)

W = weight of missile

γ = weight density of liquid
($\gamma = 62.4$ lb/ft^3 for water at 80°F)

γ_m = weight density of the missile

x = depth of missile c.g. below the initial c.g., as shown in figure 5-1.

A_0 = horizontal cross-sectional area of the missile (constant over Length L)

C_D = drag coefficient (given in table 5-1 or other references on fluid mechanics) which is a function of L/d, R and shape of the missile.

L = vertical length of the missile

d = characteristic dimension of the missile as shown in table 5-1.

$$R = \text{Reynolds number} = \frac{V_0 d}{\nu} \qquad (5-7)$$

ν = kinematic viscosity of the liquid
($= 0.95 \times 10^{-5}$ ft^2/sec for water at 80°F)

V_0 = initial velocity of the missile at $x = 0$ (See figure 5-1)

V = striking velocity of the missile at $x = H$ (See figure 5-1)

$$V_2 = \text{terminal velocity} = \left[g(1 - \gamma/\gamma_m)/a \right]^{1/2} \qquad (5-8)$$

Table 5-1

DRAG COEFFICIENT FOR VARIOUSLY SHAPED BODIES IN INCOMPRESSIBLE FLOW[25]

Form of Body	L/d	R	C_D
Circular disk		$>10^3$	1.12

Tandem disks,
  L = spacing
  d = diameter
	0
	1
	2
	3
		$>10^3$	1.12
			0.93
			1.04
			1.54

Rectangular plate,
  L = length
  d = width
	1
	5
	20
	∞
		$>10^3$	1.16
			1.20
			1.50
			1.95

Circular cylinder (axis ‖ to flow)
  L = length
  d = diameter
	0
	1
	2
	4
	7
		$>10^3$	1.12
			0.91
			0.85
			0.87
			0.99

Circular cylinder (axis ⊥ to flow)
  L = length
  d = diameter
	1
	5
	20
	∞
	
	5
	∞
		10^5
		
		$>5 \times 10^5$
			0.63
			0.74
			0.90
			1.20
			
			0.35
			0.33

Streamlined foil (1 : 3 airplane strut)
  L = span
  d = chord
	∞	$>4 \times 10^4$	0.07

Hemisphere: Hollow upstream Hollow downstream		$>10^3$	1.33 0.34
Sphere		10^5 $>3 \times 10^5$	0.5 0.20
Ellipsoid (1 : 2, major axis ‖ to flow)		$>2 \times 10^5$	0.07
Airship hull (model)		$>2 \times 10^5$	0.05
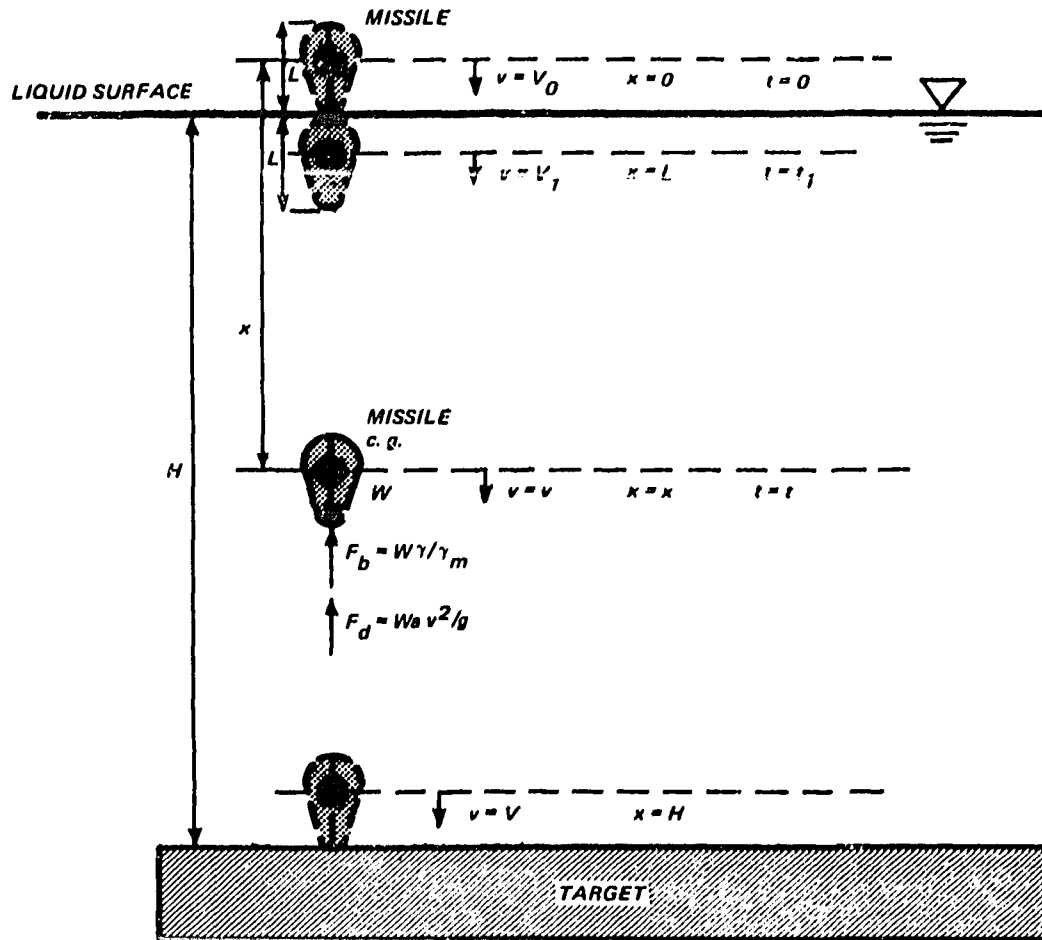
Figure 5-1

PENETRATION OF A MISSILE IN A LIQUID

APPENDIX A

CROSS REFERENCE LISTING TO AEC STANDARD SAR FORMAT

This appendix shows the cross reference between sections of AEC's Standard
SAR format and the sections of this topical report.

AEC SAR Format	BC-TOP-9
3.5.4	2.0, 3.0, 4.0

APPENDIX B

GLOSSARY

B.1   PENETRATION

Penetration is the displacement of the missile into the target. It is a measure of the depth of the crater formed at the zone of impact.

B.2   PERFORATION

Perforation is "full Penetration" or where the missile passes through the target with or without exit velocity (of missile).

B.3   SPALLING OF CONCRETE

Spalling is the peeling off of the back face of the target opposite to the face of impact.

B.4   DUCTILITY RATIO

The ductility ratio is the ratio of the maximum deflection to the deflection at the "effective yield point."

B.5   EFFECTIVE YIELD POINT

That point on an idealized bilinear resistance function separating the elastic and perfectly plastic portion of the function. The effective yield point is based on the strength of the structure by ultimate (or plastic) design methods.

B.6   ELASTIC IMPACT

An elastic collision is characterized by elastic deformations at the missile-target interface.

B.7   PLASTIC IMPACT

A plastic collision is characterized by inelastic deformation and local damage of the missile and/or target in the impact zone. For a purely plastic collision, elastic restoring forces at the missile-target interface and associated elastic rebound energy release converge to zero.

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APPENDIX C

REVIEW OF EXISTING FORMULAS

C.1   PENETRATION AND PERFORATION

The most common formulas used in determining the local effects of a missile
on a target, such as penetration, perforation, and spalling for missiles
striking either a concrete or steel target, are given in tables C-1 and C-2.
These tables include equations C-1 through C-11. These are the current
state-of-the-art formulas on impact analysis, which consists primarily of
empirical methods based on experiments conducted for specific and limited
applications. Generally, the experiments were conducted for the Government
using missiles, such as bombs and bullets, and having velocities above
1000 ft/sec. Current impact analysis assumes that the missile impinges
the target normal to the surface. The effects of the oblique angle of
striking at various velocities are illustrated in figure C-1. It can be
seen that assuming normal striking of the target is conservative, since a
small deviation from a normal impact decreases the depth of penetration
considerably.

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The Army Corps of Engineers and National Defense Research Committee
equations (table C-1) for penetration, perforation, and spalling have a
term, which depends only on the diameter of the missile. However, this
term provides overly conservative results when a low velocity and large
diameter missile is considered. For example: as $V_s \rightarrow 0$ the penetration
approaches 0.5D; perforation approaches (1.8)D; and spalling approaches
(2.8)D, which is not realistic.

Experimental data with velocities below 500 ft/sec are just beginning to
develop, with the emphasis on the effect of impact on the target. Some
experiments have been completed with missile velocities in the range of
interest. However, the tests were not necessarily conducted for target
information.[21] Therefore, available pertinent data are limited.

The modified Petry formula has had the widest application for determining
the penetration of a missile into concrete targets and is adopted for use
at the present time. It was developed by the Poncelet theory, provides
estimate of penetration, and has functioned best in the velocity range of
interest. Also, conservatism is built into this approach because of the
following:

   A.   The angle of striking the target has a large effect if the angle
        is greater than 20°. A normal angle of strike is assumed.

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B. The probability of a missile being oriented in a manner that would produce the greatest penetration is remote. In addition, any rotational effect tends to increase the area of impact.

C. Conservative estimates for weight, velocity, area of impact, and target strength provide conservatism.

Even though the modified Petry formula was developed in 1910, the material coefficient for penetration, K_p, has been revised by experiments and is reported by Amirikian[14] and shown in figure 2-1.

The BRL formula for perforation of concrete targets is used. It is selected instead of the modified Petry formula of T = 2X because the BRL formula was developed for perforation and not as an approximation from a penetration.

The BRL equation, given in equation (2-3), has been modified to account for concrete strength other than 3000 psi by replacing the constant coefficient 7.8 by $427/\sqrt{f_c'}$ in equation (C-7).[3]

Two steel perforation formulas are available, the Ballistic Research Laboratories (BRL) formula[2][3] and the Stanford Research Institute formula, known as the Stanford Equation.[20] The Stanford Equation is based on experimental data, using missile velocities within the range of interest. However, its limits of applicability are very restrictive because most missiles encountered fall outside the range of the Stanford Equation.

The Ballistic Research formula, table C-2, is used with an assigned value of K equal to unity. Rearranging terms and solving directly for T leads to the formula for calculating the threshold of perforation.

$$T = \frac{\left(\frac{MV_s^2}{2}\right)^{2/3}}{672D} \qquad (C-12)$$

The Stanford Equation (table C-2) has the following defined limits of applicability:

0.1 < T/D < 0.8,

0.002 < T/L < 0.05,

10 < L/D < 50,

5 < W/D < 8,

8 < W/T < 100,

$70 < V_s < 400$,

L = length of cylindrical missile

V_s = striking missile velocity normal to the target surface for the threshold of perforation (ft/sec)

Solving equation (C-11) directly for plate thickness gives,

$$T = \sqrt{0.045 \frac{W_m V_s^2}{DE} + 0.0022 \left(\frac{W}{W_s}\right)^2} - 0.047 \frac{W}{W_s} \qquad \text{(C-13)} \quad \Big| \, 2$$

where,

$$E = \frac{W_m V_s^2}{2g}$$

W_m = weight of missiles (pounds)

A parametric study comparing the BRL formula and the Stanford Equation, within the limits of applicability of the Stanford Equation, showed the BRL and SRI formula are generally in good agreement for the shorter spans. But, for longer spans the SRI formula is less conservative. Considering this and the narrow band of limits for the SRI equation the BRL equation is used for design. $\Big| \, 2$

C.2   MULTIPLE ELEMENT BARRIER EQUATION

Equation (2-9) assumed the residual kinetic energy of the missile after perforation (E_r) is the difference between the kinetic energy of the missile before impact (E_k) and the energy required to perforate the steel (E_p)

$$E_r = E_k - E_p = \frac{MV_r^2}{2} = \frac{MV_s^2}{2} - \frac{MV_p^2}{2} \qquad \text{(C-14)}$$

where

M = mass of the missile $\left(\dfrac{\text{lb-sec}^2}{\text{ft}}\right)$

Solving for V_r

$$V_r = \left(V_s^2 - V_p^2 \right)^{1/2}$$

This equation[27] neglects the mass of the plug which may be punched out of the target, which would be very small for a steel target; for a concrete target, the concrete would fracture and not act in conjunction with the missile mass.

V_p can be obtained from equations (2-3) and (2-7) by solving for V_s, which will be the velocity to just perforate, V_p, when a given thickness of target, t, is used.

Table C-1

CONCRETE PENETRATION, PERFORATION, AND SPALLING FORMULAS (Sheet 1 of 3)

Identification	Formula	Remarks	Equation No.
A. Penetration into Reinforced Concrete			
Modified Petry (Refs. 13, 14, 15)	$X = 12K_p A_p \log_{10} \left(1 + \dfrac{v_s^2}{215,000} \right)$	For infinitely thick slab	C-1
	$X_1 = \left[1 + e^{-4\left(\frac{t}{X} - 2\right)} \right] X$	Depth of penetration for slabs with finite thickness. $X_1 \to X$ when $t \to 3X$	
Army Corps of Engineers and National Defense Research Committee (Refs. 13, 16, 17)	$X = \dfrac{282}{\sqrt{f'_c}} \dfrac{W \, D^{0.215}}{D^2} \left(\dfrac{v_s}{1000} \right)^{1.5} + 0.5D$		C-2
Ammann & Whitney (Refs. 18, 19).	$X = \dfrac{282 \, N \, W \, D^{0.2}}{\sqrt{f'_c} \quad D^2} \left(\dfrac{V_s}{1000} \right)^{1.8}$		C-3
B. Concrete Thickness to be Just Perforated			
Modified Petry (Refs. 13, 14, 15)	$T = 2X$	X is obtained from Equation (C-1)	C-4

Table C-1

CONCRETE PENETRATION, PERFORATION, AND SPALLING FORMULAS (Sheet 2 of 3)

Identification	Formula	Remarks	Equation No.
B. Concrete Thickness to be Just Perforated (Con't)			
Army Corps of Engineers (Refs. 13, 16)	$T = 1.35D + 1.24X$	X is obtainted from Equation (C-2)	C-5
National Defense Research Committee (Refs. 13, 17)	$T = 1.23D + 1.07X$	X is obtained from Equation (C-2)	C-6
Ballistic Research Laboratories (Modified) (Ref. 13)	$T = 7.8 \ \dfrac{W}{D^{1.8}} \left(\dfrac{V_s}{1000}\right)^{1.33}$	For $f'_c = 3000$ psi	C-7
	$T = \dfrac{427}{\sqrt{f'_c}} \dfrac{W}{D^{1.8}} \left(\dfrac{V_s}{1000}\right)^{1.33}$	For any value f'_c Ref. 3	
C. Concrete Thickness to be Just Spalled			
Army Corps of Engineers (Refs. 13, 16)	$T_s = 2.2D + 1.35X$	X is obtained from Equation (C-2)	C-8
National Defense Research Committee (Refs. 13, 17)	$T_s = 2.28D + 1.13X$	X is obtained from Equation (C-2)	C-9

2|

2|

Table C-1

CONCRETE PENETRATION, PERFORATION, AND SPALLING FORMULAS (Sheet 3 of 3)

W = Weight of Missile (lb.)

V_S = Striking Velocity of Missile (ft/sec.)

D = Diameter of Missile (in.)

A_p = $\dfrac{\text{Missile Weight}}{\text{Projected Frontal Area of Missile}}$ (psf)

X = Depth of Penetration into Slab of Infinite Thick Concrete (in.)

X_1 = Depth of Penetration into a Finite Thickness Slab of Concrete (in.)

t = Thickness of the Slab (in.)

f'_c = Compressive Strength of Concrete (psi)

K_p = Experimentally Obtained Material Coefficient for Penetration (See Figure 2-1)

N = Nose Factor = $0.72 + 0.25 (n - 0.25)^{1/2}$

n = $\dfrac{\text{radius of nose section}}{\text{diameter of missile}}$

T = Thickness To Be Just Perforated (in.)

T_S = Thickness To Be Just Spalled (in.)

NOTE: Some of the equations have been rewritten to reflect consistent units and terminology.

Table C-2

PERFORATION IN STEEL FORMULAS

Identification	Formula	Remarks	Equation No.
Ballistic Research Lab (Refs. 2, 3, 13)	$T^{3/2} = \dfrac{0.5 \ MV_s^2}{17,400 \ K^2 D^{3/2}}$		C-10
Stanford Research Institute (Ref. 20)	$\dfrac{E}{D} = \dfrac{S}{46,500} \left(16,000 \ T^2 + 1,500 \dfrac{W}{W_s} T \right)$	See Limits page C-3	C-11

T = steel thickness to be just perforated (in.)

M = mass of the missile (lb-sec^2/ft),

V_s = striking velocity of the missile normal to target surface (ft/sec),

K = constant depending on the grade of the steel, (K is usually = 1,)

D = diameter of the missile (in.)

E = critical kinetic energy required for perforation (ft-lb),

S = ultimate tensile strength of the target minus the tensile stress in the steel (psi)

W = length of a square side between rigid supports (in.),

W_s = length of a standard width (4 in.). (See Ref. 20)
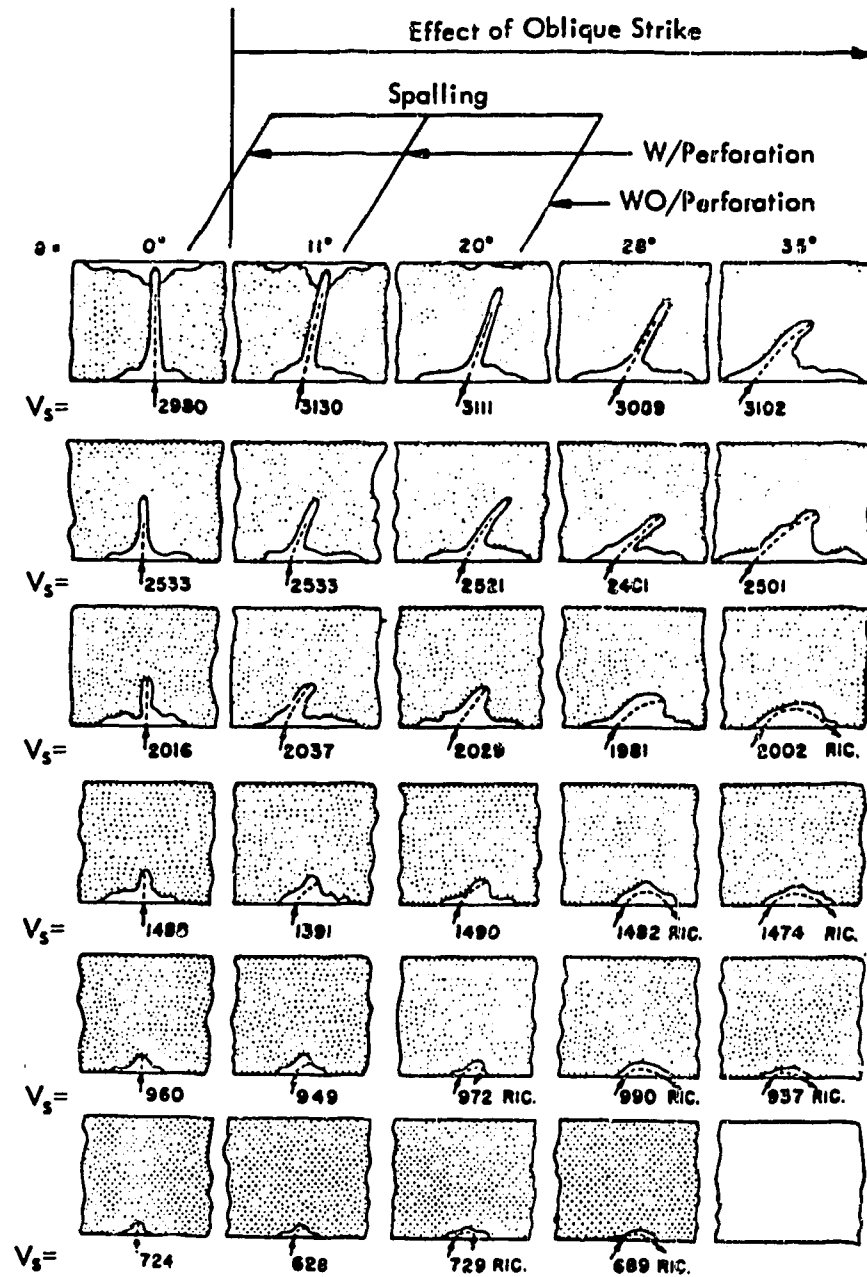
37 MM. M80 Projectile
Concrete Thickness = 22". Compressive strength = 5700 lbs/in.^{2}
Striking velocity (V_s) and angle of obliquity (θ) shown.
Stuck projectiles and path of ricochet projectiles shown.

Figure C-1

TYPICAL CRATER PROFILES

APPENDIX D

DERIVATIONS

D.1   DERIVATION OF FORCE-TIME HISTORY FOR AUTOMOBILE CRASH, EQUATION (5-1)

An approximate relationship has been observed in experiments on automobile crashes.[22] The deceleration per unit deformation associated with the crushing force was observed to be approximately the same for a wide variety of standard-size U.S. automobile makes and models. The deceleration during a frontal impact is as follows:

$$- \ddot{x} = 12.5g \; x \qquad \text{(D-1)}$$

where

$-\ddot{x}$ = deceleration (ft/sec^2)

x = distance automobile crushes into target (ft)

g = gravitational acceleration (ft/sec^2)

Newton's law of motion and equation (D-1) give the relation

$$F = - \frac{W_m}{g} \ddot{x} = 12.5 \; W_m \; x \qquad \text{(D-2)}$$

where

W_m = weight of automobile (lb)

Equation (D-1) is the motion for an undamped linear oscillator with a unit mass and a spring constant equal to 12.5g. Its solution with initial zero deformation is

$$x = C \sin (12.5g)^{1/2} \; t \qquad \text{(D-3)}$$

To determine the constant, C, consider the balance of the input kinetic energy, E_m, by the striking automobile with work done by the impact force plus energy lost, E_L, by other phenomena such as target response

$$E_m = \frac{1}{2} \frac{W_m V_s^2}{g} = \frac{1}{2} F_{max} \; x_{max} + E_L \qquad \text{(D-4)}$$

where

V_s = striking velocity of the automobile (ft/sec).

D-1

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In the conservative case of $E_L = 0$ the constant C can be determined by substituting equations (D-2) and (D-3) into equation (D-4)

$$C = \left(\frac{1}{12.5g}\right)^{1/2} V_s \qquad (D-5)$$

Finally substituting equations (D-3) and (D-5) into equation (D-2) gives the force-time history

$$F = 12.5 \ W_m \left(\frac{1}{12.5g}\right)^{1/2} V_s \ \sin \ (12.5g)^{1/2} \ t$$

$$= 0.625 \ V_s \ W_m \ \sin \ 20 \ t \qquad (D-6)$$

This is a sine wave of frequency $\omega = 20$ rad/sec and period $T = 2\pi/\omega$ = 0.314 sec. The maximum force occurs at $t = T/4 = 0.0785$ sec when the velocity of the striking automobile is zero relative to the rigid surface and then rapidly reducing to zero. Thus under the condition of plastic collision (i.e., missile and target acquire same velocity after impact) the duration of the impact force is from $t = 0$ to $t = T/4 = 0.0785$ sec. At $t = 0.0785$ sec., the force diminishes from a maximum value to zero.

As an example of using the resulting expressions, consider the experimental data in reference 23. Test No. 505-IW for a 1963 Plymouth automobile striking a rigid wall yielded the following data.

$$W_m = 3270 \ \text{lb}$$

$$V_s = 53.3 \ \text{mph} = 78.17 \ \text{ft/sec}$$

$$x_{max} = 3.82 \ \text{ft}$$

$$gF_{ave}/W_m = 25g$$

(average over distance)

From equations (D-3) and (D-5) and the above data the stopping distance is

$$x_{max} = \left(\frac{1}{12.5g}\right)^{1/2} (78.17) = 3.91 \ \text{ft}$$

According to the forcing function equation (D-6) the average deceleration (average over distance, not over time) for Test No. 505-IW is

$$gF_{ave}/W_m = gF_{max}/2W_m = (0.625)(78.17)g/2 = 24.42g$$

which agrees with the test result (25g) quite closely.

D.2 DERIVATION OF THE VELOCITY OF A MISSILE AFTER IT HAS PENETRATED THROUGH A LIQUID

Consider the motion of a missile, length L, entering a liquid medium and striking a target at depth H from the liquid surface, as shown in figure 5-1. When the missile first hits the liquid, a compressive shock wave may be generated in the liquid with a resulting loss of missile velocity. This is called the "compression phase" of liquid entry in reference 24, (page 18). As the missile displaces the liquid it experiences a hydrodynamic force with variable impact drag coefficient C_p. This "liquid-displacement phase" further reduces the velocity. After the maximum missile cross-sectional area is immersed, the "cavity drag phase" is initiated in which the drag coefficient C_D may be considered constant. In this appendix the velocity of the missile during liquid entry is analyzed on the assumptions that the velocity loss in the "compression phase" is negligible and that the impact drag coefficient C_p in the "liquid-displacement phase" is equal to the drag coefficient C_D in the "cavity drag phase." Since C_D is always smaller than C_p (see reference 24, page 30 and figure 2-7) these assumptions give more conservative (high) results for the missile velocity. Only the case of vertical entry (normal to the horizontal liquid surface) is considered.

Under these assumptions, the equation of missile motion is

$$\frac{W}{g}\ddot{x} = W - F_b - F_d \tag{D-7}$$

where

W = Weight of missile

g = gravitational acceleration

x = depth of missile c.g. below the initial c.g. as shown in figure 5-1

t = time after initial contact of missile with liquid

F_b = buoyant force

F_d = drag force

and a dot denotes differentiation with respect to t.

Between x = 0 and x = L the buoyant force varies with x

$$F_b = \gamma \int_0^x A(x_1) \, dx_1 = \gamma f(x), \quad (0 \le x \le L) \tag{D-8}$$

where

γ = weight density of the liquid

$A(x_1)$ = horizontal cross-sectional area of the missile at vertical distance x_1 from the tip .

When $x > L$ the buoyant force is a constant

$$F_b = W\gamma/\gamma_m, \quad (x > L) \tag{D-9}$$

where

γ_m = weight density of missile

The drag force is given by the expression

$$F_d = \gamma \, A_m C_D v^2/2g \tag{D-10}$$

where

A_m = maximum horizontal cross-sectional area of missile

$v = \dot{x}$ = velocity of missile at depth x

If the liquid is assumed to be incompressible, the drag coefficient, C_D, in equation (D-10) is a function of the missile shape and the Reynolds number R, defined as

$$R = \frac{V_0 d}{\nu} \tag{D-11}$$

where

d = characteristic dimension of missile as shown in table 5-1

V_0 = initial velocity (at t=0 and x=0) of missile

ν = kinematic viscosity of liquid

Table 5-1 from reference 25 lists some typical values of C_D for variously shaped bodies in incompressible fluid flow. Reference 24 (page 35) presents some C_D values for a family of nose shapes. Other references on fluid mechanics can also be consulted.

Substituting equations (D-8), (D-9) and (D-10) into equation (D-7) results
in the following two forms of the equation of motion and solutions:

A.   For $0 \leq x \leq L$

$$\ddot{x} + a\dot{x}^2 + bf(x) - g = 0 \ , \ (0 \leq x \leq L) \tag{D-12}$$

where

$$a = \gamma A_m C_D / 2W \tag{D-13}$$

$$b = \gamma g / W \tag{D-14}$$

and $f(x)$ is given in equation (D-8).

This is a nonlinear, second order, nonhomogeneous, ordinary differ-
ential equation for $x(t)$.

According to reference 26 (page 551) it can be solved as follows:

Let

$$y(x) = \dot{x}^2 = v^2 \tag{D-15}$$

Then if a prime denotes differentiation with respect to x,

$$y'(x) = 2\dot{x}(\dot{x})' = 2\dot{x} \ \ddot{x}/\dot{x} = 2 \ \ddot{x} \tag{D-16}$$

Equation (D-12) becomes

$$y'(x) + 2ay(x) = 2g - 2bf(x) \tag{D-17}$$

which is a linear, first order, nonhomogeneous, ordinary differen-
tial equation for $y(x)$, and has the solution

$$y(x) = \left\{ 2 \ \int \mu(x) \ [g-bf(x)] \ dx + c \right\} \Big/ \mu(x) \tag{D-18}$$

where c is the integration constant and

$$\mu(x) = e^{\int 2adx} = e^{2ax} \tag{D-19}$$

Substituting equation (D-19) into equation (D-18) gives

$$y(x) = v^2 = e^{-2ax} \left[2g \int e^{2ax} dx - 2b \int e^{2ax} f(x) dx + c \right]$$

$$= g/a - 2be^{-2ax} G(x) + c\, e^{-2ax}, \quad (0 \le x \le L)$$

(D-20)

where

$$G(x) = \int e^{2ax} f(x) dx = \int e^{2ax} \left[\int_0^x A(x_1) dx_1 \right] dx \quad \text{(D-21)}$$

in which equation (D-8) has been used.

At the initial position (See figure 5-1) $x = 0$, $v = V_0$, and equation (D-20) gives

$$c = V_0^2 - g/a + 2bG(0) \tag{D-22}$$

Then equation (D-20) becomes

$$y(x) = v^2 = g/a + e^{-2ax} \left\{ V_0^2 - g/a \right.$$

(D-23)

$$\left. + 2b \left[G(0) - G(x) \right] \right\}, \quad (0 \le x \le L)$$

At $x = L$ equation (D-21) gives

$$G(L) = \left\{ \int e^{2ax} \left[\int_0^x A(x_1) dx_1 \right] dx \right\}_{x=L} \tag{D-24}$$

and equation (D-23) gives

$$y(L) = V_1^2 = V_2^2 + g\gamma/\gamma_m a + e^{-2aL} \left\{ V_0^2 - g/a \right.$$

(D-25)

$$\left. + 2b \left[G(0) - G(L) \right] \right\}$$

where V_1 is the missile velocity at $x = L$ (See figure 5-1) and

$$V_2^2 = \frac{g}{a}\left(1 - \gamma/\gamma_m\right)$$ (D-26)

2

Consider the special case of a missile with uniform horizontal cross-sectional area A_0. Then $A(x_1) = A_0$. Equation (D-21) gives

$$G(x) = \int e^{2ax}\left(\int_0^x A_0 dx_1\right)dx = A_0\int xe^{2ax}dx$$ (D-27)

$$= A_0 e^{2ax}(2ax-1)/4a^2 , \quad (0 \le x \le L)$$

2

from which

$$G(0) = -A_0/4a^2$$ (D-28)

and

$$G(L) = A_0 e^{2aL}(2aL-1)/4a^2$$ (D-29)

Equation (D-23) becomes

$$v^2 = g/a + bA_0(1 - 2ax)/2a^2 + e^{-2ax}\left(v_0^2 - g/a - bA_0/2a^2\right), \quad (0 \le x \le L)$$ (D-30)

Formulas for other missile shapes can be derived similarly.

For $x \ge L$

$$\ddot{x} + a\dot{x}^2 + g\gamma/\gamma_m - g = 0 , \quad (x \ge L)$$ (D-31)

This is a special case of equation (D-12) with

$$f(x) = g\gamma/\gamma_m b , \quad (x \ge L)$$ (D-32)

which, when substituted into equation (D-20), gives

$$v^2 = V_2^2 + ke^{-2ax} , \quad (x \ge L)$$ (D-33)

The integration constant k can be determined by the condition that at $x = L$, $v = V_1$ obtained in equation (D-25)

$$k = \left(V_1^2 - V_2^2\right) e^{2aL} \tag{D-34}$$

Hence the missile velocity at $x \geq L$ is given by

$$v = \left[V_2^2 + \left(V_1^2 - V_2^2\right) e^{-2a(x-L)}\right]^{1/2} , \quad (x \geq L) \tag{D-35}$$

Substituting V_1 from equation (D-25) into equation (D-35) gives

$$v = \left\{V_2^2 + e^{-2ax} \left[2b \ (G(0) - G(L)) + V_0^2 \right. \right.$$
$$\left. \left. + g \left(e^{2aL} \ \gamma/\gamma_m - 1\right) /a\right]\right\}^{1/2} , \quad (x \geq L) \tag{D-36}$$

In the special case of a missile with uniform horizontal cross-sectional area A_0 equations (D-28) and (D-29) are substituted into equation (D-36) to give

$$v = \left\{V_2^2 + e^{-2ax} \left[bA_0 \left(e^{2aL} (1 - 2aL) - 1\right)/2a^2 \right. \right.$$
$$\left. \left. + V_0^2 + g \left(e^{2aL} \ \gamma/\gamma_m - 1\right)/a\right]\right\}^{1/2} , \quad (x \geq L) \tag{D-37}$$

At $x = H$, when the missile strikes the target (See figure 5-1) the velocity V is given by equation (D-36) or equation (D-37) with x replaced by H.

APPENDIX E

SAMPLE APPLICATIONS

E.1   CONCRETE (PENETRATION, PERFORATION AND SPALLING)

A 4-inch x 12-inch wooden plank, weighing 108 pounds, strikes at 300 mph
(440 fps) in a normal head-on collision with a reinforced concrete
(f'_c = 3000 psi) wall. The plank has a 48 square inch cross-sectional
area with the equivalent diameter of 7.8 inches.

E.1.1   PENETRATION

Penetration is given by equation (2-1):

$$X = 12 \ K_p \ A_p \ Log_{10} \left(1 + \frac{v_s^2}{215000} \right)$$

For 3000 psi concrete K_p = 0.00348 (figure 2-1)

and

$$A_p = \frac{108}{48/144} = 324 \ psf$$

Then

$$X = 12 \times 0.00348 \times 324 \times Log_{10} \left(1 + \frac{440^2}{215000} \right) = 3.77 \ in.$$

When the thickness of a wall is less than 3 x 3.77 = 11.3 in., the depth of
penetration is given by equation (2-2):

$$X_1 = \left[1 + e^{-4\left(\frac{t}{X} - 2\right)} \right] X$$

For example, for a wall with thickness t = 8 in., we get:

$$X_1 = \left[1 + e^{-4\left(\frac{8}{3.77} - 2\right)} \right] \times 3.77 = 6.08 \ in.$$

E-1

E.1.2 PERFORATION

The thickness of a wall to be just perforated is given by formula 2-3:

$$T = \frac{427}{\sqrt{f'_c}} \frac{W}{D^{1.8}} \left(\frac{V_s}{1000}\right)^{1.33}$$

For $f'_c = 3000$ psi,

$$T = \frac{427}{\sqrt{3000}} \frac{108}{7.8^{1.8}} \left(\frac{440}{1000}\right)^{1.33} = 7.01 \text{ in.}$$

Therefore, the concrete thickness required to prevent perforation according to equation 2-4 is:

$$t_p = 1.25 \times 7.01 = 8.76 \text{ in.}$$

E.1.3 SPALL

The thickness of a wall to be just spalled is given by equation (2-5).

$$T_s = 2 \, T = 2 \times 7.01 = 14.02 \text{ in.}$$

Therefore, the concrete thickness required to prevent spalling according to equation (2-6) is:

$$t_s = 1.25 \times 14.02 = 17.53 \text{ in.}$$

E.2 STEEL TARGETS

Given: A ten pound missile one inch in diameter impacts a target at 200 ft/sec.

Question: Find the thickness of steel plate, T, to just perforate and the thickness t_p required to prevent perforation.

Solution: Use equation (2-7) and (2-8)

Then

$$T = \frac{\left[\frac{10}{2 \times 32.2} (200)^2\right]^{2/3}}{672(1)} = 0.5 \text{ inches}$$

and $t_p = 1.25 \times 0.5 = 0.625$ inches.

E.3 STRUCTURAL RESPONSE

Consider a 10 lb solid metal missile of 1-inch diameter striking with 200 ft/sec velocity at the mid-span of a simply-supported steel I-beam of 10 ft span and AISC designation W6x12[8] with static yield strength f_y = 50,000 psi. It is required to evaluate the structural response of the beam according to Section 3 under the condition of plastic impact.

According to equation (3-17) the effective mass of the steel beam may be conservatively estimated as the mass of a 13-inch length of the beam (since the depth of beam d = 6" and D_x = 1", the missile diameter) which is for W6x12 beam[8],

$$M_e = \frac{(12)(13)}{12g} = 13/g$$

According to equation (3-8) for plastic impact, the required target strain energy to absorb the impact energy is

$$E_s = \frac{M_m^2 V_s^2}{2\left(M_m + M_e\right)} = \frac{\left(\frac{10}{g}\right)^2 (200 \times 12)^2}{2\left(\frac{10}{g} + \frac{13}{g}\right)} = 32,440 \text{ in.-lb}$$

The resistance-displacement function of a simply-supported beam under central loading can be idealized as a bilinear function (figure 3-1 and table 4-2) with

$$R_m = \frac{4M_u}{L} = \frac{8If_{dy}}{Ld} = \frac{8(21.7)(50,000)(1.2)}{(10 \times 12)(6)} = 14,467 \text{ lb}$$

and

$$x_e = \frac{R_m L^3}{48EI} = \frac{(14,467)(10 \times 12)^3}{48(30 \times 10^6)(21.7)} = 0.80 \text{ in.}$$

where the value of the moment of inertia, I, for the beam cross-section is taken from reference 8, and modulus of elasticity E = 30 x 10^6 and dynamic increase factor DIF = 1.2 (table 4-1) have been used.

According to figure 3-1 the maximum strain energy for purely elastic structural response is

$$E_e = \frac{1}{2} R_m x_e = \frac{1}{2}(14,467)(0.80) = 5,787 \text{ in.-lb}$$

which is less than E_s = 32,440 in.-lb, so the structural response is elasto-plastic. Then according to equation (3-22) the required ductility ratio is

$$\mu_r = \frac{E_s}{x_e R_m} + \frac{1}{2} = \frac{32,440}{(0.80)(14,467)} + 0.5 = 3.30$$

Since, according to table 4-4 the allowable ductility ratio for a steel beam under lateral loads is 20, this beam can withstand the postulated missile impact if no other loads are acting simultaneously. In case other loads are present as missile impacts and remain in effect throughout the structural response the required ductility ratio should be evaluated by equation (3-24) instead of equation (3-22).

E.4   MISSILE PENETRATION THROUGH WATER

Consider the postulated accident condition of a fuel shipping cask (the missile) falling from an overhead crane and possibly damaging the spent fuel pool floor slab (the target) underneath. The cask is a cylinder with length L = 17 ft, diameter d = 7 ft, and weight W = 2 x 10⁵ lb. [The spent fuel pool contains water of depth H = 37 ft. If the cask is to drop h = 11 ft to just hit the water surface the initial velocity is

$$V_0 = (2\ gh)^{1/2} = \left[2(32.17)\ (11)\right]^{1/2} = 26.6\ \text{ft/sec.}$$

The Reynolds number is, according to equation (5-7),

$$R = \frac{V_0 d}{\nu} = \frac{(26.6)\ (7)}{0.93 \times 10^{-5}} = 2.0 \times 10^7.$$

Since L/d = 17/7 = 2.43 the drag coefficient is, according to table 5-1 for the case of circular cylinder with axis parallel to flow and with R > 10³,

$$C_D = 0.854$$

The horizontal cross-sectional area is

$$A_0 = \pi d^2 / r = \pi(7)^2 / 4 = 38.5\ \text{ft}^2.$$

Then equation (5-5) gives

$$a = \frac{\gamma C_D A_0}{2W} = \frac{(62.4)(0.854)(38.5)}{2\ (2 \times 10^5)} = 0.0051\ \text{ft}^{-1},$$

and equation (5-6) gives

$$b = \frac{\gamma g}{W} = \frac{(62.4)(32.17)}{2 \times 10^5} = 0.010 \text{ ft}^{-1} \text{ sec}^{-1}.$$

The weight density of the cask is

$$\gamma_m = \frac{W}{A_0 L} = \frac{2 \times 10^5}{(38.5)(17)} = 305.6 \text{ lb/ft}^3.$$

According to equation (5-8) the terminal velocity is

$$V_2 = \left[g \left(1 - \gamma/\gamma_m \right)/a \right]^{1/2}$$

$$= \left[(32.17)(1 - 62.4/305.6)/0.0051 \right]^{1/2} = 70.9 \text{ ft/sec.}$$

Since H > L, and according to equation (5-2)

$$Z_2(L) = V_2^2 + e^{-2aL} \left\{ \frac{bA_0}{2a^2} \left[e^{2aL}(1 - 2aL) -1 \right] \right.$$

$$+ V_0^2 + \frac{g}{a} \left(e^{2aL} \gamma/\gamma_m -1 \right) \Bigg\} =$$

$$(70.9)^2 + e^{-2(0.0051)(17)} \left\{ \frac{(0.01)(38.5)}{2(0.0051)^2} \left[\right. \right. \quad {}^{.734} (1 - 0.1734)$$

$$\left. - 1 \right] + (25.6)^2 + \frac{(32.17)}{0.0051} \left(e^{0.1734} 62.4/305.6 - 1 \right) \Bigg\} =$$

$$5027 + (0.8408) \left[- 4193 \right] = 1502 > 0,$$

the value of $Z_2(H)$ should be calculated:

$$Z_2(H) = V_2^2 + e^{-2aH} \left\{ bA_0 \left[e^{2aL}(1 - 2aL) - 1 \right]/2a^2 + V_0^2 \right.$$

$$\left. + g \left(e^{2aL}\gamma/\gamma_m - 1 \right)/a \right\} = (70.9)^2 + e^{-2(0.0051)(37)}(-4193) =$$

$$5027 + (0.6856)(-4193) = 2152 > 0$$

Finally the striking velocity of the cask on the spent fuel pool floor slab is, according to equation (5-4).

$$V = \left[Z_2(H) \right]^{1/2} = (2152)^{1/2} = 46.4 \text{ ft/sec}$$

It is interesting to note that if the spent fuel pool is dry the striking velocity would be

$$V = \left[2g (h + H) \right]^{1/2} = \left[2 (32.17)(11 + 37) \right]^{1/2} = 55.6 \text{ ft/sec}$$

For missiles of lighter weights, the reduction of striking velocity due to the presence of a liquid would be more pronounced.

APPENDIX F

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