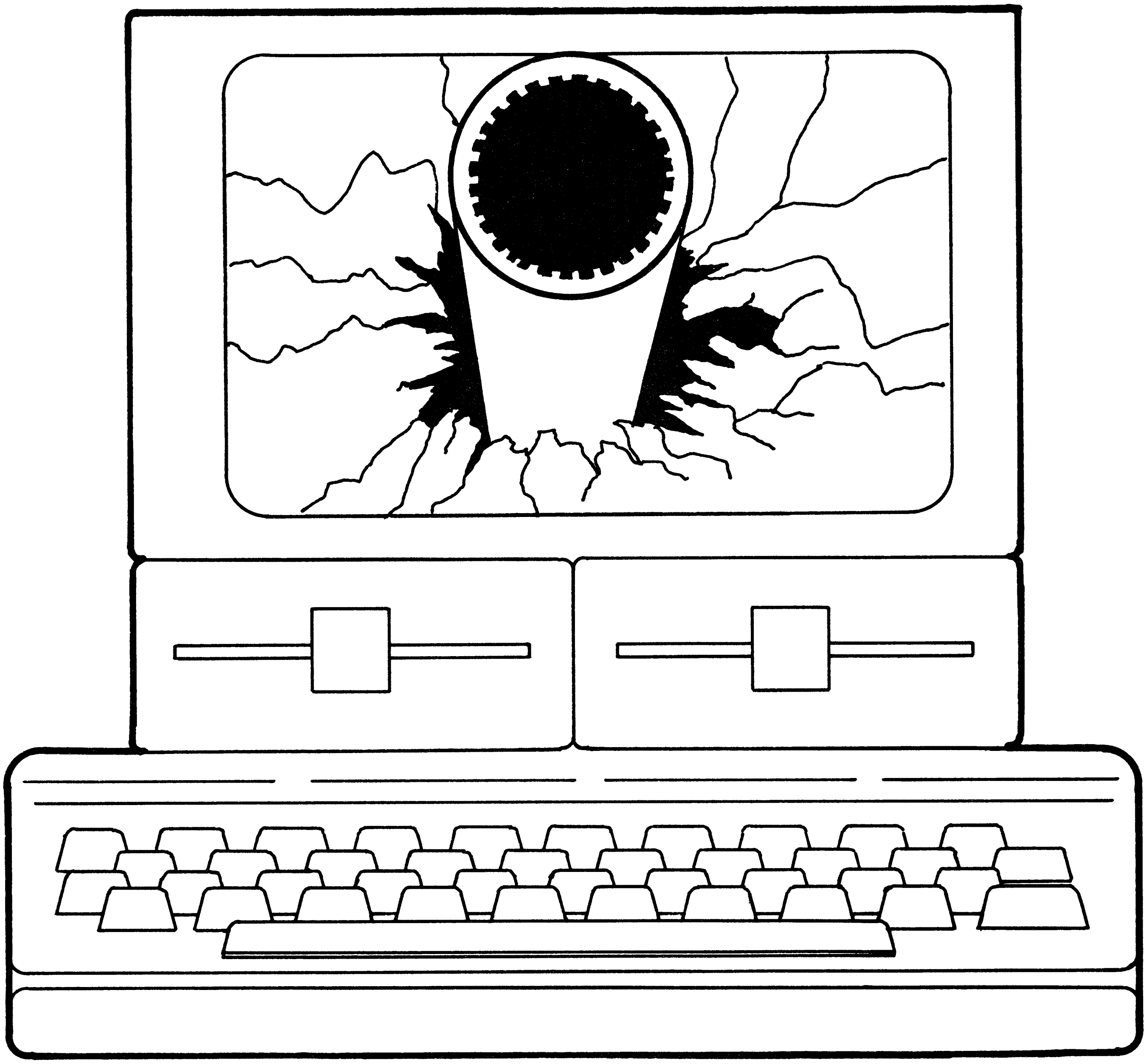


Exterior Ballistics with MICROCOMPUTERS

by W. R. Jurens

ROY L. NAVY HAS A THEORY concerning the loss of HMS *Hood*, but cannot confirm it without knowing the ballistics of the *Bismarck*'s rifles. Jack Numbercruncher, having carefully studied Nathan Okun's now classic articles on armor penetration, is ready to compute the immunity zones for Austro-Hungarian battleships, but lacks ballistic information. Larry Longdrop wants to estimate the effects of aerial bombing on various ship designs, but is unfamiliar with the computational methods to employ. Despite the fact that gun systems lately have been almost entirely replaced by the missile as the primary weapons system of most surface warships, the fact remains that the naval rifle was considered to be one of the major arbiters of ship to ship combat for the majority of the time period of interest to readers of this journal. It is therefore somewhat surprising that until now there has been virtually nothing published concerning the detailed characteristics of these vital components of the naval arsenal. In spite of certain recent and commendable efforts to shed at least tabular light on the subject, it is still true that specific details on the armament systems installed aboard the ships being considered have been either heavily glossed or completely ignored in recent publications on ship design and construction.

The grand problem of the ballistician can be simply stated as follows: given a projectile of known physical properties such as size, shape, and mass, projected from a rifle at a known initial velocity (the determination of which is a problem in INTERIOR ballistics), the task is to compute the remaining velocity, angle of fall, time of flight, and range at any point on the resulting trajectory under specified atmospheric conditions. In the absence of other information, the official 'range table' produced by the country in question must be considered the definitive definition of the ballistics of any gun weapons system. Sadly, in a large percentage of cases, official range tables remain classified, inaccurate, or frankly non-existent. Although it has always been possible in principle to recompute or create from scratch those tables that were otherwise unavailable, it was only with the advent of the small, reasonably priced microcomputer that such a path became practically feasible. This paper is intended to present what is believed to be the first generally available microcomputer program for exterior ballistics investigations in naval history, accompanied by a sufficient theoretical base to allow the prospective user to generate accurate and complete range tables for any reasonable gun weapon system he or she might wish to study.

The formal study of exterior ballistics in Western culture most likely began in Greek or Roman times when it became valuable to be able to predict the fall of shot of such weapons as catapults and ballistae. Considering the rudimentary measuring instruments in use at the time, the embryonic state of complex mathematics, and the virtually complete ignorance of atmospheric physics and its effects on projectile motion, it is no surprise that early attempts at predicting the trajectory of a projectile were unsuccessful. Although the best physical and mathematical minds in Europe up to the time of Galileo and Newton attempted to solve the ballistic problem, it was not until the early part of the 20th century that an inkling of the entire process which was required to accurately compute the simple trajectory of a projectile was understood and could be practically applied to problems in the field. Even then, the required calculations—while not in and of themselves difficult—were extremely laborious to perform in any quantity. Accordingly, specialized approximation techniques, such as the Siacci and Ingalls methods, (still used by those interested in small-arms ballistics) were used wherever possible. Although these methods proved adequate for trajectories with angles of departure under about 15 degrees, the introduction of much improved fire control systems between the world wars made battle ranges previously considered impossible not only attainable but likely. This, together with the requirement for extensive and complete trajectories for anti-aircraft fire, made the computations of large numbers of high-angle trajectories a necessity—a process which could only be effectively completed by the use of the high-speed digital computer. It is no coincidence that E.N.I.A.C., the Electronic Numerical Integrator And Calculator, perhaps the very first 'computer' worthy of the name, was for the first few years of its life engaged almost exclusively in the computing of firing tables for the U.S. Army and Navy. Today, the owner of any one of the small microcomputers commonly available, or even one of the more powerful programmable calculators, has at his disposal a device likely a hundred times more capable than E.N.I.A.C., at under one-thousandth the cost.

The solutions to ballistic problems have increased steadily in accuracy and complexity over time. All discussions to follow incorporate certain assumptions which, while rendering the solutions almost useless for actual gunnery in the field, are inconsequential in historical analysis. These are:

A) There is no wind.
B) A 'standard' atmosphere is assumed, where the variation of atmospheric characteristics with height obeys well-defined mathematical laws.
C) The earth is flat and non-rotating.
D) The projectile is always tangent to the trajectory (i.e. is non-yawed), and is non-spinning.
E) The dimensions of the rifle and target are considered trivial with respect to the trajectory as a whole.
F) The acceleration of gravity is a constant.
G) The rifle is stationary at the time of firing.

It is well to note that these assumptions are implicit in the construction of most historical range tables as well.

Vacuum Trajectories

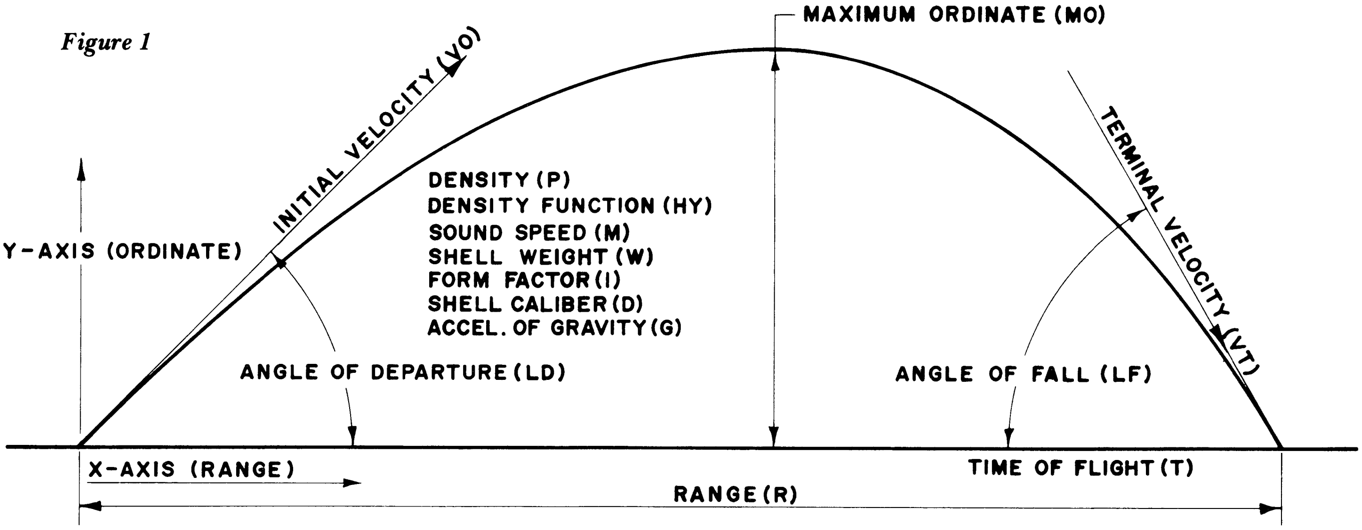
Although the calculations required to completely predict the trajectory of a projectile in a resisting medium such as air are extremely complex, the same computations in the absence of air, or 'in vacuo,' constitute a rather trivial problem in elementary mathematics; one that is, in fact, taught in most high school physics courses. Even though the application of the 'in vacuo' formulae are of little use in solving practical problems of gunnery, they nonetheless can give a very fair estimate of a weapon's maximum potential performance if the initial velocity is relatively low, the projectile relatively dense, and the angle of departure is large. A study of the characteristics of vacuum trajectories also serves as an excellent 'jumping-off' point for more complex concepts, and so such a study is an excellent place at which to begin. Those unfamiliar with the terminology commonly used in ballistics should refer to figure 1.

Neglecting atmospheric drag, the trajectory takes the form of a parabola with its characteristics dependent only upon the initial velocity of the projectile, VO; the angle of departure, LD; and the value assigned to the acceleration of gravity, g. The equations are:

$$\text{RANGE} = X = \frac{VO^2}{g} \sin 2\,LD$$

$$\text{MAX. ORDINATE} = MO = \frac{VO^2}{2g} \sin^2 LD$$

Figure 1



$$\text{TIME OF FLIGHT} = T = \frac{2\,VO}{g} \sin LD$$

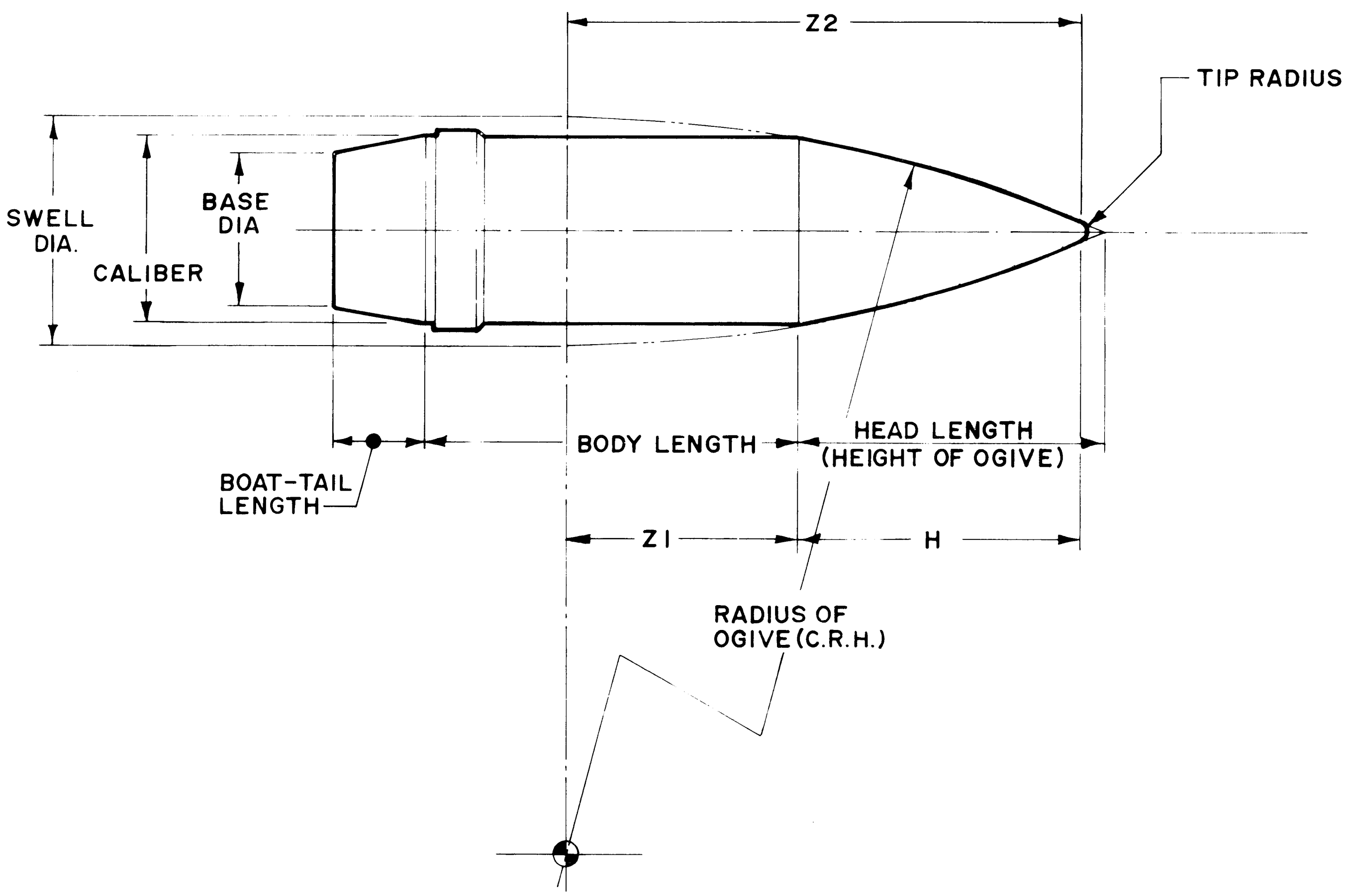
$$\text{TERMINAL VELOCITY} = VT = VO$$

$$\text{ANGLE OF FALL} = LF = LD$$

In summary then, when air resistance is neglected, the angle of fall is equal to the angle of departure, the terminal velocity is equal to the initial velocity, the maximum ordinate is equal to one-half the total range, and the shape and weight of the shell are unimportant. As an exercise, the reader might wish to confirm that assuming VO = 330 M/sec, LD = 32 degrees, and g = 9.80 M/sec^2, the terminal trajectory conditions are VT = 330 M/sec, LF = 32 degrees, range X = 9987.6 meters, and time of flight T = 35.688 seconds.

The Ballistic Coefficient

Although the simplifications involved in the previous method would be adequate if we were fighting a war in space, or conducting a naval engagement on the moon, in real life it is essential to consider air resistance in ballistic computations if useful results are to be obtained. Ninety-five percent of the effort involved in trajectory computation reflects the need to take atmospheric effects into consideration. To begin our study



of atmospheric ballistics, we must begin by introducing one of the most fundamental concepts of the ballistician's art, the ballistic coefficient, C.

Almost anyone who has observed the path of an object thrown through the air has observed, at least unconsciously, that the distance the object travels is strongly affected by both its weight and its shape. This difference in 'carrying power' is what makes it impossible to throw a paper wad as far as a baseball, even though their shape and size might be virtually identical. The baseball travels much farther than the paper wad because its weight (or what is almost the same thing, its density) is much greater. This variation in range due to density is handled in formal ballistics by the ballistic coefficient C, which can simply be written:

$$C = \frac{W}{i\,D^2}$$

where W equals the mass of the projectile in kg, i equals a form factor used to correct for small variations in projectile geometry, and D equals the diameter of the projectile in m. Assuming i = 1.00 for now, the ballistic coefficient of a 400 mm diameter 1000 kg bullet would be equal to 1000 $(1.00 \times .40^2)$ or 6250. Although strictly speaking the ballistic coefficient is no longer in use by ballisticians (insofar as they converted to the terminology and techniques of aerodynamicists in the 1960s), for the purposes of this discussion, the concept remains perfectly adequate, and is easier for the general reader to understand. A full discussion of the (rather arbitrary) differences between the old and new computational techniques is given in NOTS TP 3902, AMCP 706-242, and Hoerner.

The Drag Function, Kd

In order to complete our determination of the ballistic coefficient, C, we must obtain and utilize the appropriate value for i, the form factor. This requires understanding of the drag function Kd, to which i is related. (See note 1).

The drag function is a graphic presentation or numerical tabulation of the resistance or deceleration of a projectile due to air resistance, measured over a wide range of velocities. Each projectile shape has a characteristic and unique drag function, but in practice it is usually assumed that the projectile of interest has a drag function curve proportional to one for a 'standard' projectile, corrected by means of the form factor, i. When expressed graphically, the drag function is usually presented as shown in figure 2, in which the ordinate or vertical axis of the graph gives the drag coefficient, Kd, (in more modern terminology CDo), and the horizontal axis of the graph, the abscissa, is graduated in units of Mach number, where Mach 1 is equal to the speed of sound. The original drag function (commonly known as the 'Gavre function' or 'Kd1') was determined by the French Gavre Commission in the 1880s and represents the then popular blunt-nosed projectile shown as type 1 on figure 3. This was the most common function in use until at least the mid-1930s worldwide, although it required rather heavy correction by means of the form factor in order to remain useful. Later, other functions were determined for projectiles of more 'modern' shape such as the type 7 and type 8. The British, working in parallel with the Americans, developed their '1940 law' at the beginning of WW II and used it thereafter. Previously they had apparently used the Gavre function, or the curve shown as the '1938 law' taken from the 1938 Handbook of Ballistics and Gunnery. In simple terms, it can be seen as a correction term relating the drag coefficient of some standard projectile to that of the projectile of interest, thus compensating in a simple way for small changes in projectile geometry. For example, at Mach 2.5 the drag coefficients for the type 1 and type 8 projectiles are about .212 and .112 respectively. Thus the form factor for a type 8 projectile vs type 1 AT THAT PARTICULAR MACH NUMBER would equal .112/.212 or .528; i.e. its drag would only be about half as great as the older shell. Because the drag function curves for type 1 and type 8 are not simply related to each other, the form factor changes with changes in projectile velocity, thus complicating the matter; nonetheless it can be used as a valuable approximation in the absence of better information. The shapes of some of the common standard projectiles that generated the drag function curves on figure 2 are shown on figure 3, with dimensions in calibers.

For any given velocity, it can be shown that the retardation of the projectile in meters/sec² is equal to $(Kd\ i\ p\ V^2)/C$ where Kd equals the drag coefficient of the bullet at the particular velocity in question (taken from figure 2), p equals the air density in kilograms per cubic meter, V equals the velocity in meters per second, and i equals the form factor. Once the retardation is known, then the motion of the projectile in air can be determined to virtually any required degree of accuracy by incorporating this value into the computation for a vacuum trajectory in steps sufficiently small that the non-linearity of the drag function over time becomes negligible. This procedure of small steps coupled with successive and converging error correction is the essence of the numerical integration technique to be discussed later. Although other methods, such as those developed by Siacci or Ingalls, can achieve comparable accuracy under a severely limited range of initial conditions, when the computations are being done by computer rather than by hand, the time savings obtained in using these simplified methods is relatively small and they are rarely used today.

The form factor was long used as a method to adjust the computed trajectory to match the results of actual field firings. Thus it can be used as a sort of mathematical 'catch-all,' or 'fudge factor' to compensate for the effects of virtually all unknown or neglected factors in computational procedure. In the absence of

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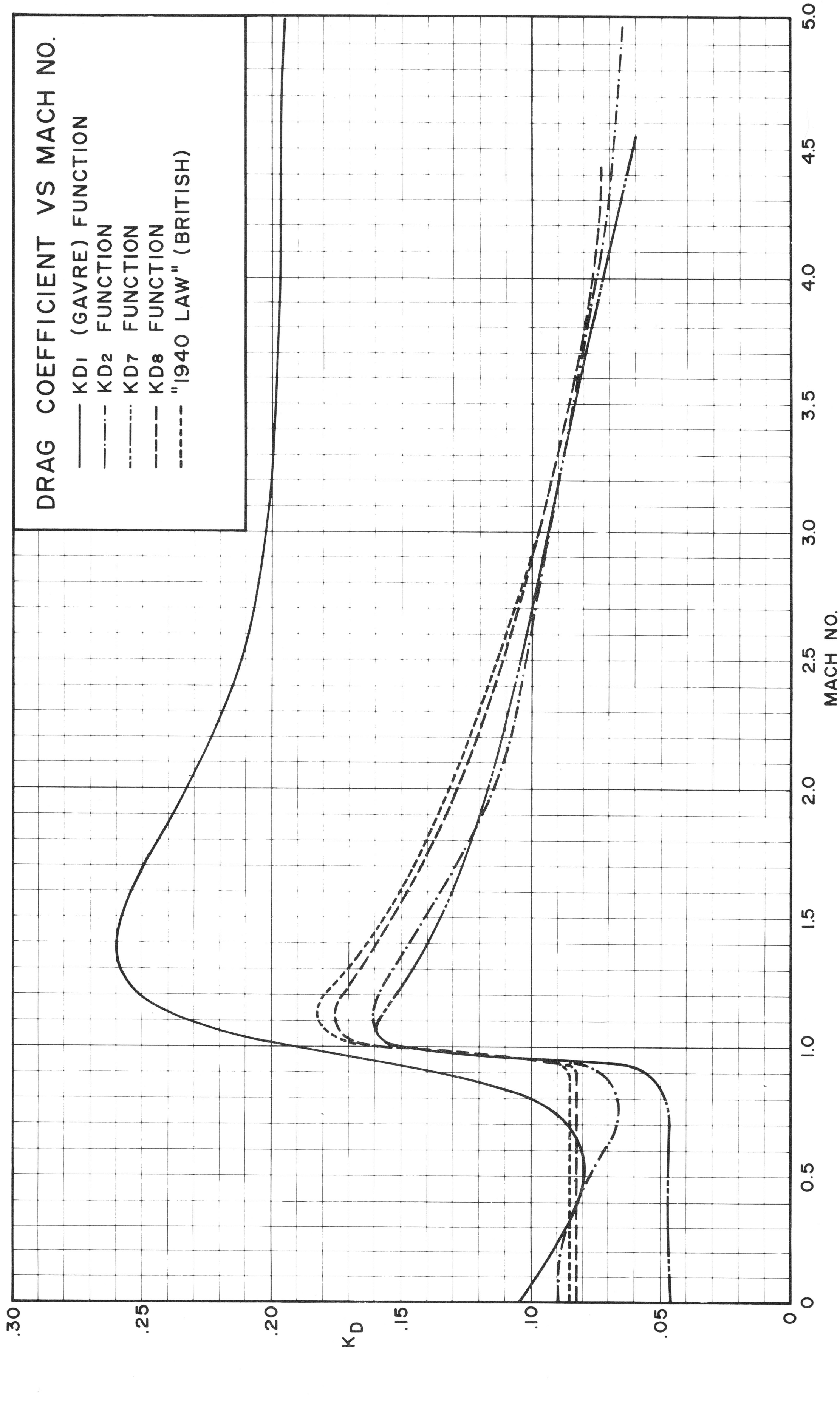
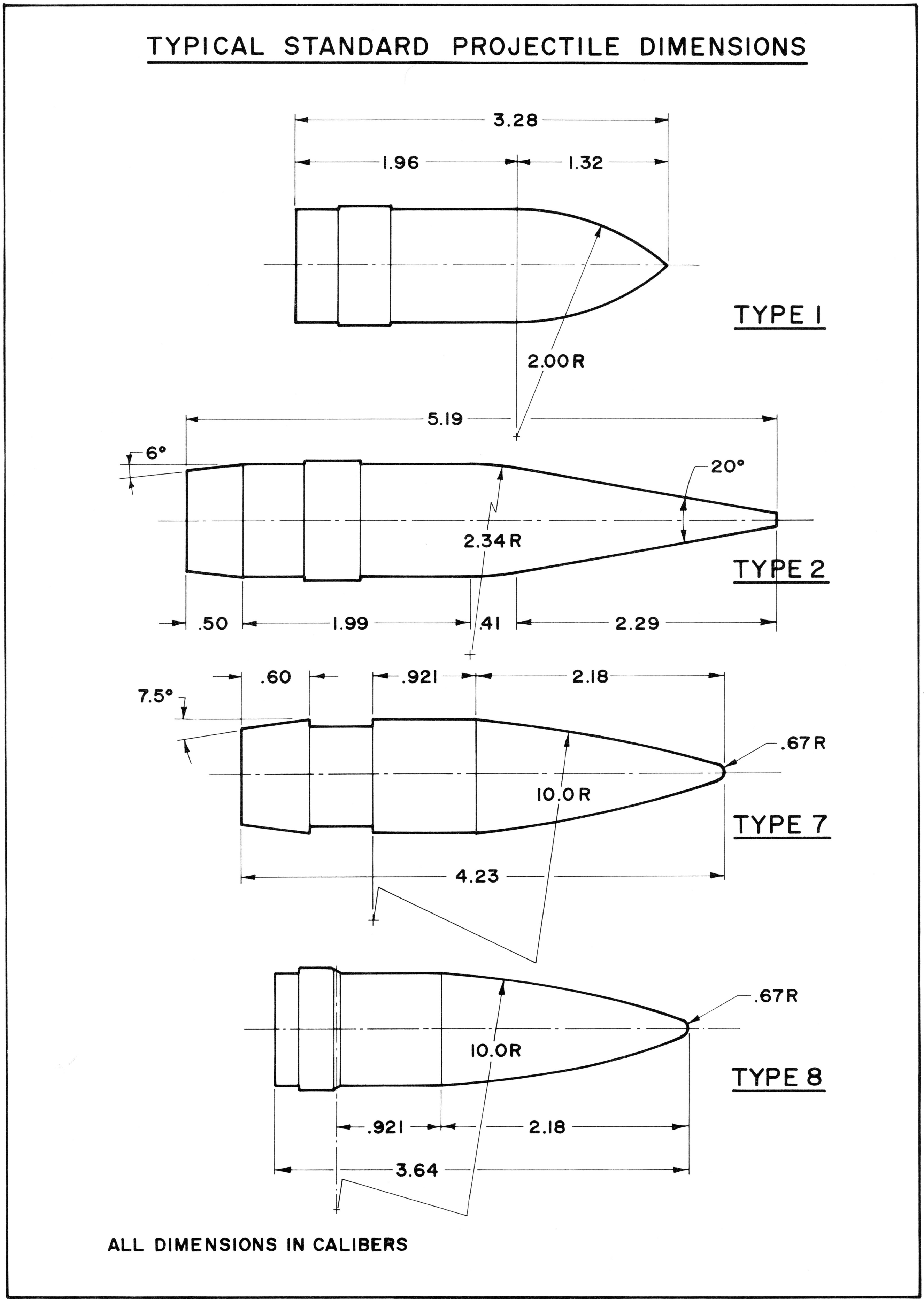
Note 1: As mentioned above, this discussion has been done in the conventional "ballistic" system of units popular until the early 1960s. Most current computation is done in the newer "aerodynamic" system, where the drag coefficient is defined as Cd, or occasionally, Cdo. For reference, Cd = Cdo = 2.546 Kd, and conversely, Kd = .3927 Cd or Cdo. The system of units in use in any particular document can usually be obtained from the date, the context, the labels on the graphs, or by inspection, as typical values in the aerodynamic system are about three times larger than those in the ballistic system.

Figure 2


DRAG COEFFICIENT VS MACH NO.
KD1 (GAVRE) FUNCTION
KD2 FUNCTION
KD7 FUNCTION
KD8 FUNCTION
"1940 LAW" (BRITISH)
MACH NO.
KD
0
.05
.10
.15
.20
.25
.30
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0



TYPICAL STANDARD PROJECTILE DIMENSIONS
3.28
1.96
1.32
TYPE 1
2.00R
5.19
6°
20°
2.34R
TYPE 2
.50
1.99
.41
2.29
.60
.921
2.18
7.5°
.67R
10.0R
TYPE 7
4.23
.67R
10.0R
TYPE 8
.921
2.18
3.64
ALL DIMENSIONS IN CALIBERS

Figure 3

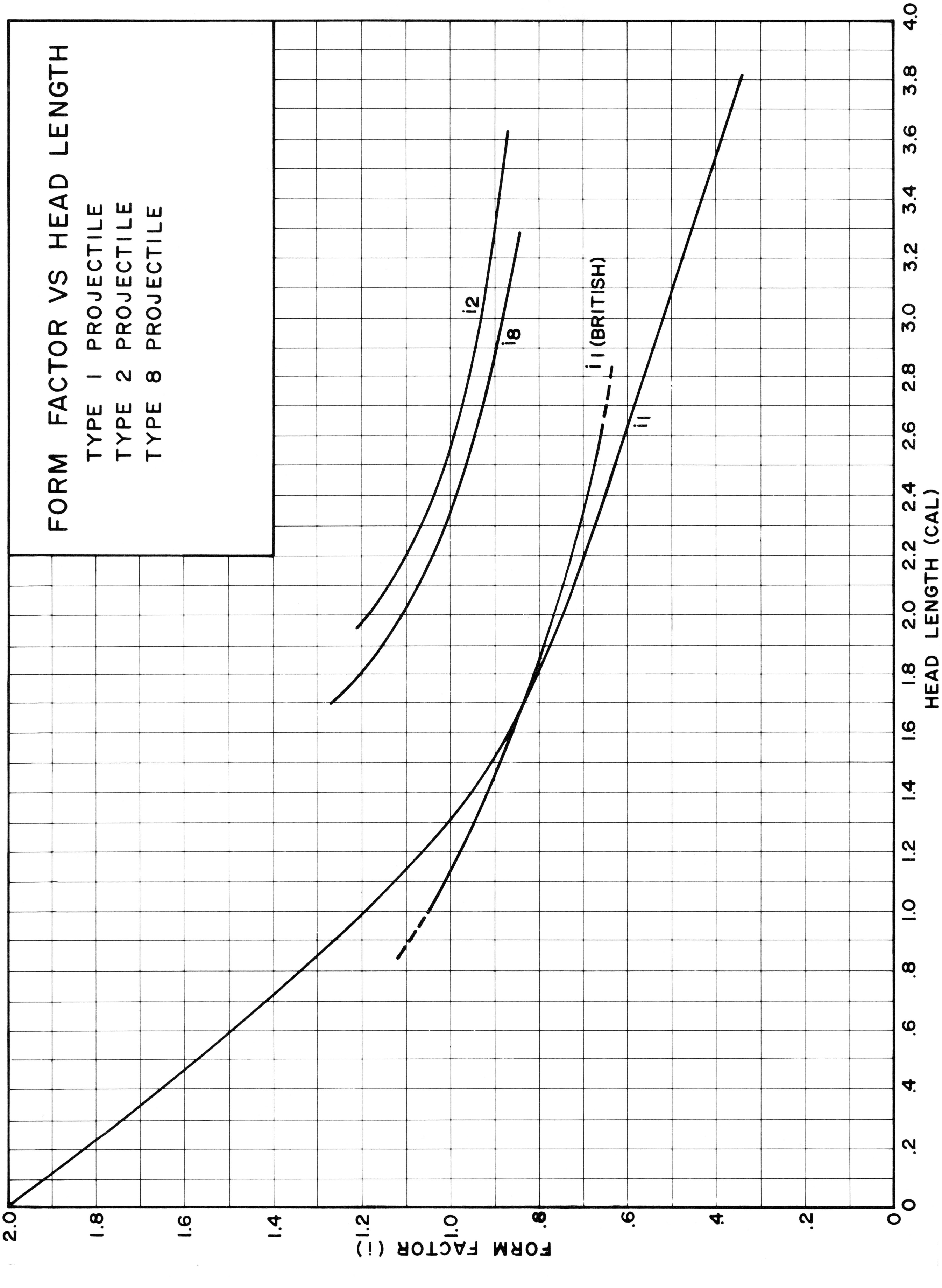

FORM FACTOR VS HEAD LENGTH
TYPE I PROJECTILE
TYPE 2 PROJECTILE
TYPE 8 PROJECTILE
i2
i8
i1 (BRITISH)
i1
HEAD LENGTH (CAL)
FORM FACTOR (i)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0

Figure 4

actual field results to calibrate the form factor, which will be the case in the majority of problems of interest here, there are a number of emperical equations and guidelines that can be used to estimate it directly 'off the drawing board.' Some of the more important of these are given below:

Effect of Head Length & Ogival Radius:

As shown in figure 3, most spin-stabilized artillery ammunition can be seen to be made up of a roughly cylindrical after section, the body, blended more or less evenly into a tapering nose section, or ogive. Tangent ogive designs, such as the type 1 projectile, have a perfect blend, whereas the so called 'secant ogive' designs, such as type 8, are discontinuous at the joint. A projectile such as type 1 where the nose is struck with a 2 calibers radius arc is described as having a '2 Calibers Radius Head,' or "2 C.R.H." In a standard ogival headed projectile where the ogive meets the head tangentially, the head length, i.e. the distance between the theoretical tip of the nose of the projectile and the point where the shell becomes cylindrical, measured down the projectile's centerline, can be shown to be:

$$HL = (N - .25)^{.5}$$

and conversely, the calibers radius head is given by the relationship:

$$N = HL^2 + .25$$

where N equals the number of calibers radius head and HL equals the head length in calibers. Using this equation, a 7 C.R.H. tangent ogive will have a head length of 2.598 calibers and vice-versa. If the ogive in use is not tangent, but is struck in such a way as to leave a shoulder near the bourrelet, as for example in projectile type 8, then the equation does not directly apply. Projectile type 8 is said to have a "5/10 C.R.H.," meaning that the ogive is struck with a radius of 10 calibers, but that the head length is only 2.18 calibers, equivalent to that of a 5 C.R.H. projectile. The confusion is increased by the fact that at times projectiles were called up as though they were tangent ogive designs although this was not the case, as in the British 4.7 in./62 lb. shell which had a 5/10 C.R.H. but a 'service nomenclature' of 6 C.R.H., or the 15 in./1938 lb. practice projectile which was also nominally a 6 C.R.H., but was in actuality a 4.52/10. Caution must therefore be used in assuming that a published figure was an actual one in the absence of an engineering drawing, dimensioned sketch, or carefully scaled photograph. Further, many projectiles have, for various reasons, small flattened areas at the nose which means that the actual head length is somewhat less than the above equations would indicate. For example, although the type 8 projectile has a 10 C.R.H. which would equate to a head length of 3.122 calibers, in actuality because of the small rounded area at the nose, and because the ogive does not meet the cylindrical portion of the shell smoothly, (i.e. it is not a tangent ogive), it has an effective head length of only 2.18 calibers as shown on figure 3. Fortunately, although head length is itself fairly important, small errors in the value chosen form the ogival radius are of relatively little consequence.

Having ascertained at least a working value for the head length, then figure 4 can be used to estimate the form factor appropriate to the given drag function. For example, if one were to use the Kd1 (Gavre) function as standard, but was interested in a 4 C.R.H. projectile instead of the 2 C.R.H. for which the Gavre function was computed, the effective form factor can be found by reading up from the 4 C.R.H. head length value, $(4 - .25)^{.5} = 1.936$), to the line marked 'i1,' and obtaining the approximate value .77. In the absence of any detailed information whatever, one can often make an educated guess as to the actual value of the ogival radius by noting that there was a steady increase in the radius over time, with 2 C.R.H. shells being common prior to and during WW I, 5–7 C.R.H. typical between the wars, and 7–10 C.R.H. being used during WW II and after; some recently manufactured extended range projectiles have an astounding 40 C.R.H.! Although the radius of curvature of the head does have an effect on the drag coefficient and drag function curve, the effect is usually rather small compared to that of head length, quite large changes in radius rarely changing the value of Kd more than ±.05.

Effect of Flat Nose:

In the absence of other data, the effect of a flat nose on projectiles having the same head length, such as might be created by the installation of a fuze, can be approximated by the formula:

$$IC = 1 + (.375\ DN^2)$$

Where IC equals the corrected form factor and DN equals the diameter of the flat nose in calibers. For example, the corrected form factor for a 400 mm caliber projectile with a 75 mm flat nose would be approximately

$$1 + (.375 \times .1875^2) = 1.013$$

Effect of Base Area and Boattail:

If the projectile is traveling under the speed of sound, that is if its velocity is less than Mach 1, the value of Kd for two otherwise similarly shaped projectiles is roughly proportionate to their relative base areas. Above Mach 1, the effect of a boattail is somewhat more difficult to evaluate, being affected by other factors such as the position and size of the driving band(s). In general, it appears best to use the Kd7 drag function curve with a form factor proportionate to the relative base areas of the unknown and the type 7 projectiles. As the type 7 projectile has a base area .709 times that of a flat based shell, the form factor of a projectile with a base area of .640 would therefore run against Kd7 with a form factor of .640/.709 = .902. Unfortunately, as the form factor changed caused by a variation in base area is not linear over Mach no., no simple correction can be entirely satisfactory. One quite accurate but tedious method would be drawn in the approximate curve for the appropriate drag function on figure 2 by using the curves for the type 7 and type 8 projectiles as a guide, as these can be considered identical except for the boattail. The new coordinates of the estimated drag function curve could then be entered into the computer as a new curve entirely, as will be described later. British practice was simply to assign a boattail projectile a form factor of .92. Incidentally, while adding a boattail might thus seem to be a relatively simple and easy way to decrease the drag and thus increase the range of a projectile at no cost, in practice the presence of a boattail tends to increase both the natural dispersion of the shell and the wear of the rifling of the gun.

Effect of Projectile Yaw

For a number of reasons that cannot be discussed here, projectiles fired from rifled guns do not exactly follow the curvature of the trajectory, but instead are almost always canted slightly above and to the right of the direction of flight. This causes, among other things, the phenomena of drift. As the magnitude of this yaw increases, the effective ballistic coefficient of the projectile similarly decreases, inasmuch as a greater projectile area (caliber) is exposed to the air without a compensating increase in mass. It is not uncommon for projectiles exiting the muzzle to experience transient yaws of up to 5 degrees, although in normal projectile designs this rapidly damps to a residual yaw (commonly called the 'yaw of repose') of two degrees or less. Basically the yaw of the bullet is determined by complex equations relating to its shape, spin rate, and moment of inertia. To complicate matters, the stability and yaw of the projectile tends to change from trajectory to trajectory as the angle of departure changes—the rapidly spinning stable shell has greater difficulty following a highly curved trajectory than a flat one and therefore its yaw is, on average, greater. Because of this effect, it is almost impossible to give more than a general estimate of the yaw magnitude of a given projectile without getting very specific, but the effect on Kd of yaw for most shells closely approximates .005 per deg^2. For example, the increase in Kd for a projectile yawed 2 degrees would be approximately $.005 \times 2^2$ or $+ .02$. A full description of the methods of compensating for yaw can be found in almost any of the newer references in the bibliography. Most ballistic computations completed before about 1940 ignored the consequences of yaw and included its effects in the form factor sump.

Atmospheric characteristics:

The drag function curves discussed earlier have for the most part been developed in 'laboratory' situations, or have been reduced to conform to a mathematically perfect atmosphere to aid in computations. As might be expected, some adjustments therefore must be made if we are to use these drag function curves under 'real life' conditions. Although it is not the purpose of this paper to go into a long discussion on the effects of meteorological conditions on trajectories, which are of little practical value to the researcher in any case, it is nonetheless necessary to briefly consider how assumed initial conditions in the laboratory atmosphere have an effect on the output of the final computed trajectory.

The atmosphere, far from being mathematically perfect, is, as any weather watcher knows, imperfect, inhomogeneous, and virtually unpredictable over time. Although the methods of compensating for changes in the weather are of great importance in practical gunnery, in theoretical ballistics all projectiles are assumed to travel through a so called 'standard atmosphere' of stable and predictable characteristics. Unfortunately, until the 1950s there was little or no international consensus as to the value of the variables to be used in the standard atmosphere, with the result that every nation chose its own values more or less arbitrarily, and changed them without notice as the situation required. Even worse, some of the earlier standard atmospheric variables were mere 'guesstimates' or values chosen primarily to suit computational expedience. A few of the more useful standard atmospheres that have been used in the past are tabulated below:

[See following page for Table]

These differences in the standard atmosphere mean that often the actual relative performance of a pair of naval rifles cannot be directly compared even if official range tables are available for both simply because the tables have been computed to apply under different conditions. In order to obtain a valid picture of the gun's relative performances, the determination of the form factor, the drag function, and the composition of the standard atmosphere must all, of necessity, be identical. Although there is usually not enough information available to reduce ALL of these factors to a common base, it is true that one of the primary advantages of the system being described here is that is does allow at least some allowance for these factors to be made, if required.

Range Tables

Though the contents and format of range tables varied from country to country and from time to time, all range tables (such as the sample U.S. range table reproduced with this article) contained values for terminal velocity, angle of fall, time of flight, maximum ordinate, and angle of departure tabulated against range, typically in 100 yard (or 100 meter) increments. In addition, most range tables could be used in conjunction with tables of differential effects that enabled quick estimation of the effects on the trajectory of non-standard conditions, such as variations in projectile weight or initial velocity. The construction of a range table was a complicated process and subject to many variations, but the following procedure is typical. First, one obtained a rifle in good (or standard) condition and a set of carefully manufactured test projectiles. After waiting for a reasonably stable and representative day, one fired groups of 5 to 10 shots at incremental angles of departure such as 2, 5, 10, 20, 30, and 45 degrees, and carefully measured the exact initial conditions for each round. The mean range to impact for each of the test groups was found and corrections were made to the range to allow for nonstandard conditions in wind, air density, and other factors. Using techniques we have already discussed, one selected an appropriate drag function and mathematically 'fired' the projectile at elevations and velocities corresponding to those used for the actual field tests. The computed range from these mathematical

TABLE OF "STANDARD" ATMOSPHERES

Country and Time Frame	*Surface Mach 1 (M/Sec.)*	*Surface Temp. (Deg. C.)*	*Surface Air Density (KG/M³)*	*Surface Pressure (MM HG.)*	*Density/ Altitude Function*
U.S.A.:					
Pre WW II	341.458	15	1.2034	750	1—.000045Y
WW II	341.458	15(?)	1.2034	750	1—.000045Y
Post WW II	340.428	15	1.225	760	Note 1
BRITAIN:					
Pre WW II	341.986	15.55	1.222	762	Note 2
WW II	(?)	(?)	(?)	(?)	(?)
Post WW II	(?)	(?)	(?)	(?)	(?)
GERMANY:					
WW II	(?)	10.5	1.245	760	(?)
JAPAN:					
WW II	(?)	(?)	(?)	(?)	(?)
INTERNATIONAL:					
Post WW II					
A.R.D.C.	340.292	15	1.2250	760	Note 3
I.C.A.O.	340.428	15	1.2250	760	Note 3
Standard Artillery Atmosphere	(?)	15	1.2034	760	(?)

Notes:

1) Unknown, but likely similar to A.R.D.C. or I.C.A.O. Standard Atmosphere.
2) $1/(10^{(.141(Y/3048))})$.
3) (MO/R*)(P/TM) where: R* = Universal Gas Constant
P = Pressure
TM = Molecular Scale Temperature
MO = 28.9644
4) Y = Altitude in Meters.

firings was forced to match the actual range by use of the form factor, and suitable mathematical routines were used to compute as many trajectories as were required to define a smooth curve for the terminal condition of interest. Using this curve, values of the terminal conditions were picked off at even intervals of range and tabulated to form the basic range table. The final step was to compute the differential effects tables by one of a number of methods, and append them to the basic range or firing table. Essentially the same procedures as described above can be used today to construct a range table for any naval weapon of historical interest, although without the test firings of course.

The Numerical Integration Technique

The numerical integration process used in the computer program is common to many areas of mathematics and science. The interested reader will be able to find numerous variations on the theme, constructed both to suit the requirements of a given problem and to more-or-less streamline the computational procedures. Although a full appreciation of the nuances of the technique requires a rather high degree of mathematical sophistication, nonetheless its basic application to the problems of ballistics requires knowledge of only elementary mathematical operations, although they must be repeated a great many times to gain a solution. The labor of completing the enormous number of calculations required, no matter how simple each might individually be, coupled with the fact that any single error in the very long sequence of operations had the potential to negate the entire result, made the process of numerical integration extremely difficult until the advent of the digital computer in the 1940s. Detailed descriptions of the technique of numerical integration, including many useful shortcuts in computation, can be found in many standard textbooks on the subject. The following protocol, somewhat simplified to aid in understanding, is adapted from one given in Heerman's *Exterior Ballistics,* 1935.

Assume a 1000 kg 400 mm caliber type 8 projectile with an initial velocity of 800 m/sec. and an angle of departure of 30 degrees. The form factor is again assumed as being 1.00, and we shall select a value of .10 seconds as our integration interval, I, which means that for a 90 second time of flight, we would need to perform the following procedure some 900 times to complete the trajectory.

Given the initial trajectory (VO) of 800 m/sec and an angle of departure (LD) of 30 degrees, the horizontal components of the velocity (XO) is VO cos LD or 692.82 m/sec and the vertical component (YO) is equal to VO sin LD or 400 m/sec. The Mach no.

M

RANGE TABLE FOR 16-INCH GUN. (FULL CHARGE)

WEIGHT OF PROJECTILE, 2100 POUNDS. INITIAL VELOCITY, 2600 F.S.
Standard powder temperature, 90°F.;
variation of ±10°F. changes the initial velocity ±20 f.s.

Range.	Change of range for variation of +10 feet per second initial velocity.	Change of range for variation of −10 pounds in weight of projectile.	Change of range for variation in density of air of −10 per cent.	Change of range for wind component in plane of fire of 10 knots.	Change of range for motion of gun in plane of fire of 10 knots.	Change of range for motion of target in plane of fire of 10 knots.	Deviation for lateral wind component of 10 knots.	Deviation for lateral motion of gun perpendicular to line of fire, speed of 10 knots.	Deviation for lateral motion of target perpendicular to line of fire, speed of 10 knots.	Change in height of impact for variation of 100 yards in sight bar.
1	10	11	12	13	14	15	16	17	18	19
Yards	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Feet.
37,000	220	12	1,600	218	213	437	142	295	437	331
37,100	221	12	1,606	219	214	440	143	296	440	334
37,200	222	12	1,612	220	215	442	144	298	442	337
37,300	223	12	1,618	222	217	445	144	300	445	339
37,400	224	12	1,624	223	218	447	145	302	447	342
37,500	225	11	1,630	225	220	450	146	304	450	345
37,600	226	11	1,636	226	221	452	147	306	452	348
37,700	227	11	1,642	228	223	455	148	308	455	350
37,800	228	11	1,648	229	224	457	148	310	457	353
37,900	229	11	1,654	231	226	460	149	312	460	356
38,000	230	11	1,660	232	227	463	150	314	463	359
38,100	231	11	1,666	233	229	466	151	316	466	362
38,200	232	11	1,672	235	231	469	152	318	469	365
38,300	233	11	1,678	237	233	472	152	320	472	368
38,400	234	11	1,684	238	235	475	153	322	475	371
38,500	235	11	1,690	240	237	478	154	324	478	374
38,600	236	11	1,696	242	239	481	155	326	481	378
38,700	237	11	1,702	243	241	484	156	328	484	382
38,800	238	11	1,708	245	243	487	157	331	487	386
38,900	239	11	1,714	247	245	490	158	334	490	391
39,000	240	10	1,720	249	247	493	159	337	493	396
39,100	241	10	1,726	251	249	498	160	341	498	402
39,200	242	10	1,732	253	252	503	161	345	503	408
39,300	243	10	1,738	255	255	509	162	349	509	415
39,400	244	10	1,744	257	258	516	163	354	516	423
39,500	245	10	1,750	259	261	524	164	359	524	432
39,600	246	10	1,756	261	264	532	165	365	532	443

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RANGE TABLE FOR 16-INCH GUN. (FULL CHARGE)

WEIGHT OF PROJECTILE, 2100 POUNDS. INITIAL VELOCITY, 2600 F.S.
Standard powder temperature, 90°F.;
variation of ±10°F. changes the initial velocity ±20 f.s.

Range.	Angle of departure=angle of elevation plus jump.		Increase in angle of departure for 100 yards increase in range.	Angle of fall.	Time of flight.	Striking velocity.	Drift.	Danger space for a target 20 feet high.	Maximum ordinate.
1	2	2a	2b	3	4	5	6	7	8
Yards	° '	Minutes.	Minutes.	° '	Seconds.	Ft.-secs.	Yards.	Yards.	Feet.
37,000	35 27.7	2,127.7	14.5	47 47	77.70	1,470	1,322.0	6	24,491
37,100	35 42.2	2,142.2	14.6	48 00	78.14	1,473	1,335.7	6	24,756
37,200	35 56.8	2,156.8	14.9	48 14	78.58	1,476	1,349.6	6	25,024
37,300	36 11.7	2,171.7	15.2	48 28	79.03	1,479	1,363.7	6	25,295
37,400	36 26.9	2,186.9	15.5	48 42	79.48	1,482	1,378.0	6	25,570
37,500	36 42.4	2,202.4	15.7	48 56	79.94	1,485	1,392.5	6	25,849
37,600	36 58.1	2,218.1	16.0	49 10	80.40	1,488	1,407.2	6	26,133
37,700	37 14.1	2,234.1	16.3	49 24	80.87	1,491	1,422.1	6	26,422
37,800	37 30.4	2,250.4	16.6	49 39	81.34	1,494	1,437.2	6	26,716
37,900	37 47.0	2,267.0	17.0	49 54	81.81	1,497	1,452.5	6	27,016
38,000	38 04.0	2,284.0	17.4	50 09	82.28	1,500	1,468.0	6	27,325
38,100	38 21.4	2,301.4	17.8	50 24	82.75	1,503	1,483.8	6	27,652
38,200	38 39.2	2,319.2	18.2	50 39	83.22	1,506	1,499.9	6	27,997
38,300	38 57.4	2,337.4	18.6	50 55	83.70	1,509	1,516.3	6	28,360
38,400	39 16.0	2,356.0	19.0	51 11	84.19	1,512	1,533.0	6	28,741
38,500	39 35.0	2,375.0	19.4	51 27	84.69	1,515	1,550.0	5	29,140
38,600	39 54.4	2,394.4	19.9	51 43	85.20	1,519	1,567.3	5	29,559
38,700	40 14.3	2,414.3	20.6	52 00	85.72	1,523	1,584.9	5	30,003
38,800	40 34.9	2,434.9	21.7	52 17	86.28	1,527	1,602.8	5	30,472
38,900	40 56.6	2,456.6	23.4	52 34	86.90	1,531	1,621.0	5	30,966
39,000	41 20.0	2,480.0	25.9	52 53	87.60	1,535	1,639.5	5	31,485
39,100	41 45.9	2,505.9	29.4	53 13	88.40	1,539	1,658.3	5	32,039
39,200	42 15.3	2,535.3	34.1	53 36	89.32	1,543	1,677.4	5	32,639
39,300	42 49.4	2,569.4	40.2	54 03	90.38	1,547	1,696.8	5	33,295
39,400	43 29.6	2,609.6	47.9	54 35	91.60	1,551	1,716.5	5	34,017
39,500	44 17.5	2,657.5	57.4	55 12	93.00	1,555	1,736.5	5	34,815
39,600	45 14.9	2,714.9	......	55 54	94.60	1,559	1,756.8	5	35,699

Two representative pages from a 1935 unclassified U.S. Navy book, **Ballistic and Range Tables.** *These pages show maximum range data for full charge 16-inch shots.*

is equal to 800/340 or 2.314. From our graph of the drag function for the type 8 projectile, we can find that the drag coefficient for the type 8 projectile at Mach 2.314 = .1188 and that therefore the initial retardation of the projectile (EO) is equal to (.1188 × 1.2034 × 800^2)/6250 or 14.643 M/sec^2. The horizontal component of the retardation (HO) is therefore 14.642 cos LD = 12.6811 M/sec^2 and the vertical component (JO) is equal to (14.642 sin LD) + g = 7.231 + 9.8067 = 17.128 M/sec^2.

Our first prediction of the horizontal velocity of the projectile (X1) is equal to the horizontal component found above minus the retardation for the integration interval we have chosen, viz. XO − (HO I) or 692.82 − (12.681 × 1) = 691.55 M/sec. Similarly, the first prediction of the vertical velocity (Y1) equals YO − (JO I) or 400 − (17.128 × .1) = 398.287 M/sec. The velocity at the end of the interval (V1) is evidently $(X1^2 + Y1^2)^{.5}$ or 798.04 M/sec., and the new "angle of departure" at the end of the interval (L1) = Atn (Y1/X1) = 29.938 degrees.

Tentatively, we might now assume that at the end of the one tenth second interval the projectile was 691.56 × .10 = 69.1 meters downrange and at an altitude of 398.287 × .10 = 39.83 meters. This would only be true if the projectile had been retarded at the rate for 800 M/sec for the entire interval. As the projectile was being retarded at a variable rate over this time period, however, this assumption would be incorrect. The human computer now finds himself in a curious 'catch 22' situation. Insofar as the retardation of the projectile is not a linear function of velocity, as is shown by the drag function plots, one cannot predict the velocity until one knows the retardation, and one cannot predict the retardation until one knows the velocity. The method by which we avoid this logical trap is at the heart of the numerical integration procedure.

As a first approximation, let us assume that the true retardation of the projectile for the first tenth second is equal to the AVERAGE of that for our initial and final velocities for the interval. Thus the mean vertical velocity for the interval (M1) can be computed to be (YO + Y1)/2 or 399.144 M/sec and the height at the end of the interval is M1 I or 39.914 meters. Knowing the approximate altitude, and knowing that the density of the air decreases with increasing height, we can now use one of the standard density altitude functions to determine the new density to be used in our computations, in this case .9958. Using the drag coefficient for the velocity at the end of the interval, and dividing our old ballistic coefficient by the density, we can find that the new total retardation (E1) is equal to 14.5325 M/sec, with a horizontal component (H1) of 14.5325 cos (L1) = 12.593 M/sec. The vertical component (J1), after including the term for the acceleration of gravity, is similarly 17.0595 M/sec.

We now have the retardation components for both the beginning and the end of the interval in question. The mean or average retardation components for the interval are therefore equal to (HO + H1)/2 for the horizontal component (H2), and (JO + J1)/2 for the vertical component (J2). The horizontal and vertical velocities at the end of the interval are then X2 = XO − (H2 I) = 691.5566 M/sec. and Y2 = YO − (J2 I) = 398.290 M/sec. respectively. The final velocity of the projectile (V2) is then equal to $(X2^2 + Y2^2)^{.5}$ = 798.051 M/sec, and the final angle is equal to atn (Y2/X2) = 29.937 degrees.

Comparing the values for the first and second predictions for the final velocity, V1 and V2, we find the difference to be very small, in this case only about .01 M/sec. If the person doing the computations felt that the differences were acceptable, he could then use the values obtained for the end of the first interval as initial values for the beginning of the second interval, and mathematically move the projectile along a step. If he felt the discrepancies were excessive, he would use the results of the first and second predictions to form yet a third prediction for the interval. Although in principle the process of successive approximation could be continued indefinitely, in practice the technique rapidly converged upon a limit beyond which no further precision could be obtained. The number of iterations required to cause an acceptable level of convergence increases as the time interval gets larger and a careful tradeoff was made in order to minimize the number of calculations required to complete the job. Nonetheless, as anyone who has attempted to follow even a broad outline of the procedures above will realize, the process was at best extremely tedious, error prone, and essentially impossible to complete by hand. It is a measure of the true power of the modern microcomputer that it can compute the trajectory of a projectile using essentially this procedure in a time usually only slightly greater than the actual time of flight.

The Structure and Use of the Program

The program reproduced here has been specifically written to run on a 48K Apple II + microcomputer system with at least one disk drive, although inasmuch as it uses few unusual commands, it should run virtually unaltered (except for screen or printer formatting instructions) on any popular microcomputer currently available. The Apple II +, though it may seem outmoded by many of the 'newer' computers on the market, has the distinct advantage of commanding an extremely large software (program) base, being extremely versatile in its application, being completely free of 'teething troubles,' and being very solidly established in the market. The program as presented is specifically designed to be easy to use, understand and modify rather than to be elegant or compact—the skilled programmer or mathematician will find numerous areas where coding can be improved to increase speed or efficiency at the expense of clarity. Normally the program takes a couple of seconds to compute an interval; this would mean that a long trajectory with a small interval could take over half an hour to run. To lessen this time a special mode is available that uses error limits to reduce this by about a factor of twenty (at a small cost in accuracy). If the programmer has access to one of the Basic language compilers currently available, these running times can be roughly halved.

Beginning at line 180, the program stores the coordinates of the drag function curve in the arrays M(x) and

K(x) where M(x) represents the Mach no. and K(x) represents the value of the drag coefficient at that particular velocity. In the program listing given, for example, the values given for M(27) and K(27) mean that a Mach no. of 2.05 the drag coefficient of this particular bullet is .231. If values of other drag functions are required they can be taken from the tabulations on page 67; an experienced programmer can easily write a subroutine to access them from disk in the interests of efficiency. The program begins by asking the user to input values for the initial conditions for the trajectory. Those for the initial velocity, angle of departure, caliber, and mass are considered to be self-explanatory. The form factor is usually set to 1.00 at the beginning of an investigation, though this will change as the work progresses. The integration interval, or time step at which we will compute the position of the projectile, is normally set to .1 to .25 seconds to begin with; the shorter the interval the greater the accuracy but the longer the time it will take to complete the computations. It is suggested that the user normally respond "Y" to the option for auto setting the intervals as this speeds up the program considerably by adjusting the integration interval to suit the user's specifications for accuracy. The normal value for the error tolerance, i.e. the difference in the final velocity of the projectile in the second and third predictions of the integration procedure, is about .03 M/sec, although as the program will on occasion exceed this value a smaller figure than is actually desired should be used. Although large convergence errors are relatively serious if they occur early in the trajectory, their effect is very greatly diminished as the shell gets closer and closer to 'home.' The air density factor is typically set to 1.00, subject to change as required to adjust for small variations in meteorological conditions. The height is usually set to 0 meters, as most trajectories are assumed to begin and end at sea level, although other values can be entered as required, a feature that is useful if the effects of bombing from airplanes are to be simulated. By setting the height equal to the altitude of the aircraft, the angle of departure to zero (for level bombing) and the initial velocity to the aircraft ground speed, the projectile is effectively converted into a bomb. As was mentioned earlier, the density of the atmosphere decreases with increasing altitude; the equation by which this decrease is approximated for ballistic purposes is the altitude density function. The program allows the user to select any one of three of the most common functions (listed in chronological order) for use in his computations, reprints all the initial conditions for reference, then halts until the key 'G,' for 'Go' is pressed. When this is done, the program automatically prints out the values for time in seconds, range in meters, height in meters, angle in degrees, and velocity in meters/second for each interval until the altitude is below sea level, whereupon it prints out the final characteristics of the trajectory as obtained by three point interpolation and halts.

The program is entirely written for data entry and manipulation in the metric system. For those users who still wish to use the Imperial system for computations, it is suggested that the program be modified to input data in Imperial units, convert it immediately to metric for use in internal operations, and convert it back to Imperial units immediately before presentation to screen or printer.

Examples of Actual Use of the Program

Although the program can be used to accurately simulate the performance of any weapon given the initial conditions of the trajectory, in general it will not match the results of actual test firings or existing range tables unless it is calibrated first. This procedure is exactly analogous to that used to construct the range table in the first place, except that instead of using test firings, we use what we know of the actual performance of the weapon as a benchmark instead. Readers should be cautioned that in running the examples below they should not expect to obtain EXACTLY the same results as those shown; first because the examples were run on an earlier version of the program than the one listed, and also because internal processing techniques vary somewhat from one microprocessor type to another; nonetheless the results should be very close to being identical, and serve perfectly well for illustration.

For a first example, let us attempt to reconstruct the range table for the U.S. 16 in./45 rifle used on the U.S. Navy's *Colorado* class battleships, which had an initial velocity of 2520 ft/sec and a projectile weight of 2240 lb. Although we will use the official range table (OP 750) as a reference, we will assume for the purposes of the exercise that we know only the maximum range of the weapon and the elevation required to reach that range—this is data fairly commonly available. The maximum range of the rifle is given as 40,600 yards at 46 deg 26 min elevation. Converted to metric units, the characteristics of the rifle and projectile become: Mass = 1016.057 kg, Initial Velocity = 768.096 M/sec, caliber = 406.4 mm, and range = 37124 Meters; the angle of elevation is of course unchanged, but should be converted to its equivalent decimal value of 46.4166 degrees. As the projectile is quite 'new,' we will assume drag function type 8, and because the range table is American, we shall use the U.S. standard altitude/density function. The 'auto set' option is used, with an error tolerance of .01 M/sec. In the absence of other information, we set the form factor and the air density factor equal to 1.00, the elevation to 0 meters, and the integration interval (which will change as the trajectory progresses) to .10 seconds.

The program is loaded into the computer and run using the values just developed; the error messages presented can be safely ignored as our tolerance was selected as being considerably below the .03 M/sec normally used. In a couple of minutes, the program halts and presents all terminal information for the trajectory, but at this stage we are only interested in the range, here about 36893 Meters. Considering that this result is obtained as a first approximation, this is a remarkably good prediction, varying from the 'official' value by only about 200 meters—less than a shiplength in many cases. If we had chosen another drag function for the projectile, the match would not have been nearly as good at this initial stage, although the form factor would be somewhat correct for this later.

As the predicted range is marginally under the official range, it appears that the real projectile was very slightly more streamlined than the one in the program —we therefore decrease the form factor a hair, say to .95, and run the whole trajectory again with all other conditions the same. The terminal range for the new trajectory is 37774 Meters, which effectively 'brackets' the official value for range. Over such a small interval the relationship of the form factor to range can be assumed a straight line function for all practical purposes, thus a rather simple linear interpolation formula (or a graphical plot) allows us to determine the actual form factor of the projectile at this particular range to be .9868. As a check, we run the trajectory again with the new form factor to obtain a range of 37116 Meters—virtually a perfect fit. The equivalent relationships of the other terminal characteristics are given below:

Comparison of Actual and Computed Values

RANGE: Computed = 37116; Actual = 37124 (ratio = .9997)
TERMINAL VELOCITY: Computed = 481.4; Actual = 495.3 (ratio = .9719)
ANGLE OF FALL: Computed = 56.391; Actual = 55.966 (ratio = 1.008)
TIME OF FLIGHT: Computed = 97.27; Actual = 96.10 (ratio = 1.012)

As can be seen, these values are in most cases within a couple of percent of the originals (see note 2).

The next step in the procedure is to mathematically 'fire' the projectile at angles of elevation of our choice until we have enough points to interpolate the final conditions for any range of interest, in this case assuming that the form factor is constant over range. Using this method over the entire range from 0 to 32000 meters, the differences between the computed and actual table rarely exceed two per cent. In real life it was often found that the form factor changed slightly with changes in the angle of elevation due to variations in projectile yaw. Recall that the drag function curves express the drag of the projectile at zero yaw, that is with the shell traveling exactly point forward, although the spin of a projectile fired from a rifled gun (perhaps paradoxically) causes it to yaw a variable amount partially determined by the curvature of the trajectory. Modern computational methods have no difficulty in taking the effects of projectile yaw into account, but the variables involved are complex, obscure, and for most projectiles designed before 1950, unknown. As a rule of thumb, for normal spin stabilized ammunition the form factor varied approximately linearly with elevation, the value of 0 degrees angle of departure being about nine-tenths of that at 45 degrees. The values for the terminal conditions obtained in the series of test firings above can be interpolated into units of equal range by any one of a number of methods—if all else fails a graph can be made and the terminal conditions picked off from that. Skilled programmers will find no difficulty in designing subroutines to do this automatically, formating the output as required.

As another example, let us examine the use of the program in simulating the effects of bombing from airplanes. During the attack on Pearl Harbor, the Japanese used high level horizontal bombers to augment their torpedo attacks on the U.S. battleships in battleship row. One of the bombs from this force struck the top of turret III on the USS *Tennessee* (BB-43) and penetrated the 127 mm. thick turret top, though fortunately (or unfortunately, depending on your point of view), it did not detonate high order. The master ballistics program can be used to attempt to determine the terminal characteristics of the bomb that did this damage. The projectiles dropped on the *Tennessee* were Type 99 no. 80 Mark 5 bombs remanufactured from obsolete 41cm Mark 5 armor-piercing bullets originally intended for *Nagato* and *Mutsu*. The weight of the bomb was 744.35 kg (excluding the tail assembly, and its maximum diameter, or 'caliber' was 410 mm; they were (apparently) dropped from Nakajima B5N2 ('KATE') aircraft from about 10000 feet. We will assume the Japanese flew at the nominal metric height of 3000 Meters, and that the velocity of the aircraft was 200 knots at the time of the drop—this is very nearly 100 Meters/sec. Total estimated weight is 800 kg, allowing for tail assembly, and, because WW II vintage bombs were usually not too streamlined, drag function Kd1 is chosen. We load the program and input variables as follows: Initial velocity = 100 M/sec, Angle of Departure = 0 degrees, Projectile Caliber = 410 mm, Projectile mass = 800 Kg, Form Factor = 1, Integration interval = .10 seconds, Auto Set Intervals = Y, Error Tolerance = .01 M/sec., Air Density Factor = 1, Altitude = 3000 Meters, Density Function = U.S. Pre-WW II Standard. After 'G' is pressed the program runs automatically, giving us the terminal conditions: Range = 2501 Meters, Altitude = 0, Terminal Velocity = 252.9 Meters/sec., and Angle of Fall = 68.42 degrees. There is little we can do to check this information, but BuShips War Damage Report #22 covers the damage

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Note 2: Readers who are ambitious at this point might wish to use the same methods as given above to attempt to match the sample range given on page 59. In this case it will be found that the applicable form function is equal to 1.10 and the computed terminal velocity at that range is 475.2 m/s against 456.3 m/s in the range table, an error (i.e. mismatch) of just over four percent. The likely reason for this apparent discrepancy is instructive, and well worth elucidating here. In the U.S. Navy of the 1930s, it was still standard practice to use the Gavre function for all trajectory computations with a form factor of about .61 to compensate for the newer more streamlined projectiles then in use. Kd1 with i of .61 is not really equivalent to Kd8, however, and thus irreducible errors occurred. It is worthwhile to note in this context that until the advent of rather advanced radar systems and projectile-borne radio transmitters during World War II, there was essentially no way to check the actual values of any of the terminal conditions of any long range trajectories, excepting range and time of flight. Other functions were in effect "taken on faith." Thus the "error" in terminal velocity might not represent an error at all in the real sense—it is most likely an artifact of small differences in mathematical technique, and in the choice of drag function and form factor.

A much closer match to the actual range table values will be found in this case by using the Kd1 form factor and, in this particular instance, a form factor of .63. For reference the bullet used in this range table was likely one of the 16 in. Mark III series, 3.55 calibers long with a 7 C.R.H. tangent ogive, a flat base, and a .166 caliber meplat at the nose.

in detail, and gives the angle of fall as about 75 degrees. This is reasonably close to the predicted estimates of our program, but not as close as we might like. There are several possible explanations for the discrepancy, assuming the program is correct. The most likely error is in our estimate of aircraft speed. The BuShips estimate of the angle of fall was apparently based on the assumption that the bomb carried away part of the after starboard yard of the mainmast during its fall. Assuming this is correct, and that the bomb was not deflected by this initial impact, if the aircraft were flying at its cruising speed of 142 knots (c. 75 M/sec.) rather than the 200 knots we used in our initial approximation, the angle of fall would become approximately 73.8 degrees, almost exactly the value required, with a terminal velocity if 244.6 M/sec.

This example was quite deliberately chosen to illustrate clearly that the value of the OUTPUT of the program is primarily defined by the known values and assumptions used for the INPUT, as the hoary old computer acronym 'GIGO' (Garbage In Garbage Out) explicitly warns. In this case, it would be irresponsible to accept the thesis as proven until more research were done into the flying and bombing characteristics of the Japanese bombers at Pearl Harbor. If this research failed to confirm our hypothesis on aircraft speed, then the source of the discrepancy must be sought (and found), elsewhere.

Graphical Methods

For those readers who are not "computer compatible" for whatever reason, a graphical method can be used to obtain trajectory solutions albeit with some loss in accuracy. A package has been prepared comprised of nine sheets plotting angle of fall and terminal velocity vs range for initial velocities of 600 M/sec to 920 M/sec at 40 M/sec intervals, and three "calibration" sheets used to determine the ballistic coefficient for the initial (known) trajectory. Let us use the chart set to obtain the terminal characteristics of the 16 in./45 rifle illustrated earlier, for a range of 24500 meters. The first step in the process is to compute the value for the common (not the natural!) logarithm of the ballistic coefficient, or, as it is abbreviated, "Log C." The easiest method to accomplish this is to use one of the calibration charts provided; in this case the one for 45 degrees nominal angle of departure is as close as we can come. Refer to figure 5. From the maximum range of 37.124 Km along the bottom, draw a vertical line up until it reaches the appropriate point in the group of velocity curves for 768 M/sec. From this point draw a horizontal line left to the ordinate of the graph to read off the value of Log C as approximately 1.23. If desired, Log C can be obtained by direct computation as follows, although the computation method will not automatically adjust for the effect of the form factor as does the graphical one. Note that as the charts were prepared from English system tables, the computation must be done on English or "Imperial" units. Our bullet weighs 2240 lbs and its caliber is 16 in. so using our previous equation for the ballistic coefficient, $C = W/i\,D^2$), the nominal ballastic coefficient is $2240/(1 \times 16 \times 16)$ or 8.75 and the log of this is .942. Note that this value assumes a form factor of 1.00, which would only be true for a type 1 projectile, for which the charts were prepared. In order to obtain a value equivalent to the 1.23 obtained in the graphical method above, a form factor must be included by estimation. In this case to obtain Log C of 1.23, the appropriate form factor would be approximately .515.

For many weapons designed in metric system countries, a solution chart will be found which corresponds exactly to the initial velocity required. As we have (deliberately) chosen an American weapon, the initial velocity is not an even value in the charts, and interpolation must be used. Proceed now to figures 6 and 7 which are the "bracketing" solution charts for metric velocities just above and below the 768 M/sec of the rifle we are using. In this illustration we will use linear interpolation to obtain terminal values; greater accuracy could be obtained by using additional solution charts to plot a curve from which actual terminal values for 768 M/sec initial velocities could be obtained by inspection.

On figure 6, draw a line vertically upward from the nominal range of interest; in this case 24.5 Km. On the sheaf of curves for terminal velocity, which begin on the left hand side of the chart at 760 M/sec, find the lines corresponding to Log C of 1.2 and 1.4, and sketch in the approximate line for 1.23 by eye, as is shown in dashed lines on the graph. From the point where this line intersects the vertical range line, proceed left to the vertical scale to read off the terminal velocity as approximately 471 M/sec. Similarly, from the sheaf of lines for angle of fall, a line can be located to indicate approximately 25.3 degrees. The reader is encouraged to follow the same process on figure 7, which has not been annotated, to obtain the terminal values of 496 M/sec terminal velocity and 22.8 degrees angle of fall. Linear interpolation between these two sets of figures gives final estimated values of 476 M/sec for terminal velocity and 24.8 degrees for angle of fall. Actual range table values are 457.2 M/sec and 26.31 degrees, so the error averages about 5 per cent. This appears to be unusually high; rather extensive checking by a correspondent in England indicates the typical prediction error to be considerably lower. Part of the error in this case is due to the simple linear interpolation scheme used, partly due to the fact that no allowance was made for the 1.5 degree discrepancy in the angle of departure during the calibration stage—the remainder is likely inherent unto the graphical system itself. Unfortunately, space restrictions prevent reproduction of the complete chart set in this article. A complete set of charts reduced to about 18 x 24 mm can be obtained from the author for $1.50 US, incuding postage and handling ($2 outside North America).

A General Conclusion

Exterior ballistics is not a simple subject. The author (and the editors) are painfully aware that a large proportion of *Warship International* readers, through no fault of their own, will find these contents tough going, and perhaps even totally unintelligible. Further, severe space limitations have resulted in inevitable

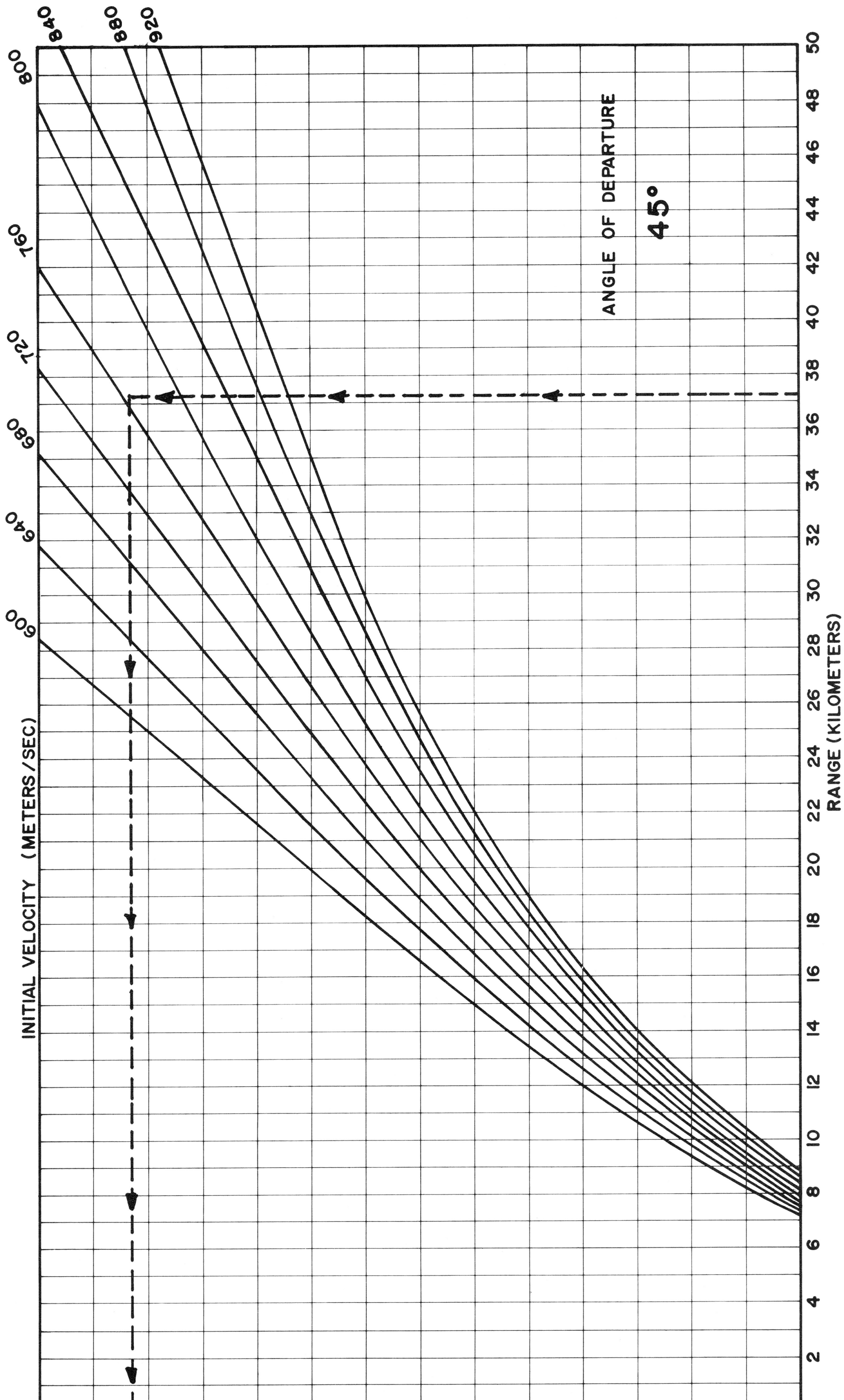
INITIAL VELOCITY (METERS/SEC)
600
640
680
720
760
800
840
880
920
ANGLE OF DEPARTURE
45°
RANGE (KILOMETERS)
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
LOG C
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 1.1 1.2 1.3 1.4

Figure 5

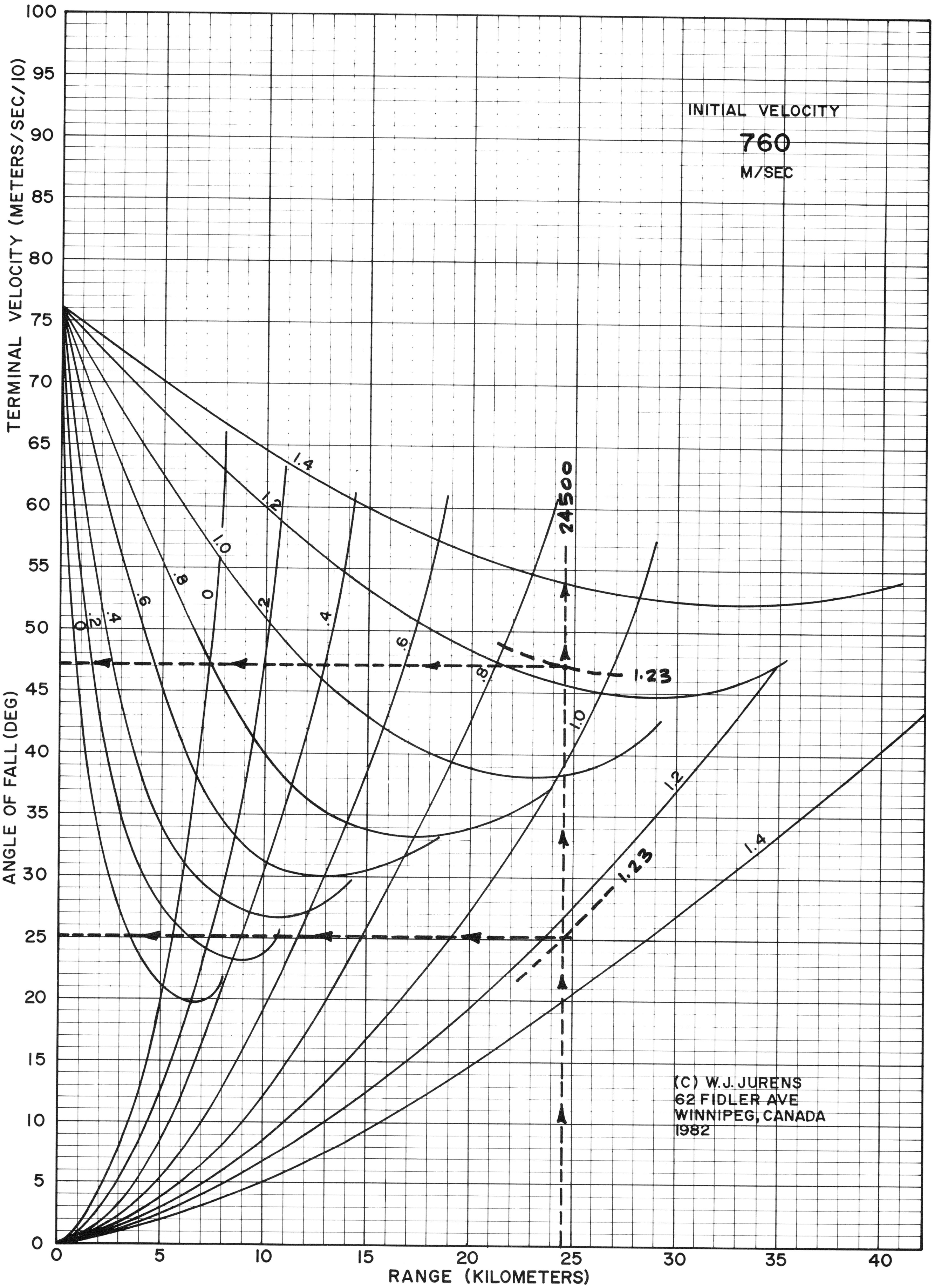
INITIAL VELOCITY
760
M/SEC
TERMINAL VELOCITY (METERS/SEC/10)
ANGLE OF FALL (DEG)
RANGE (KILOMETERS)
24500
1.23
1.23
(C) W.J. JURENS
62 FIDLER AVE
WINNIPEG, CANADA
1982

Figure 6

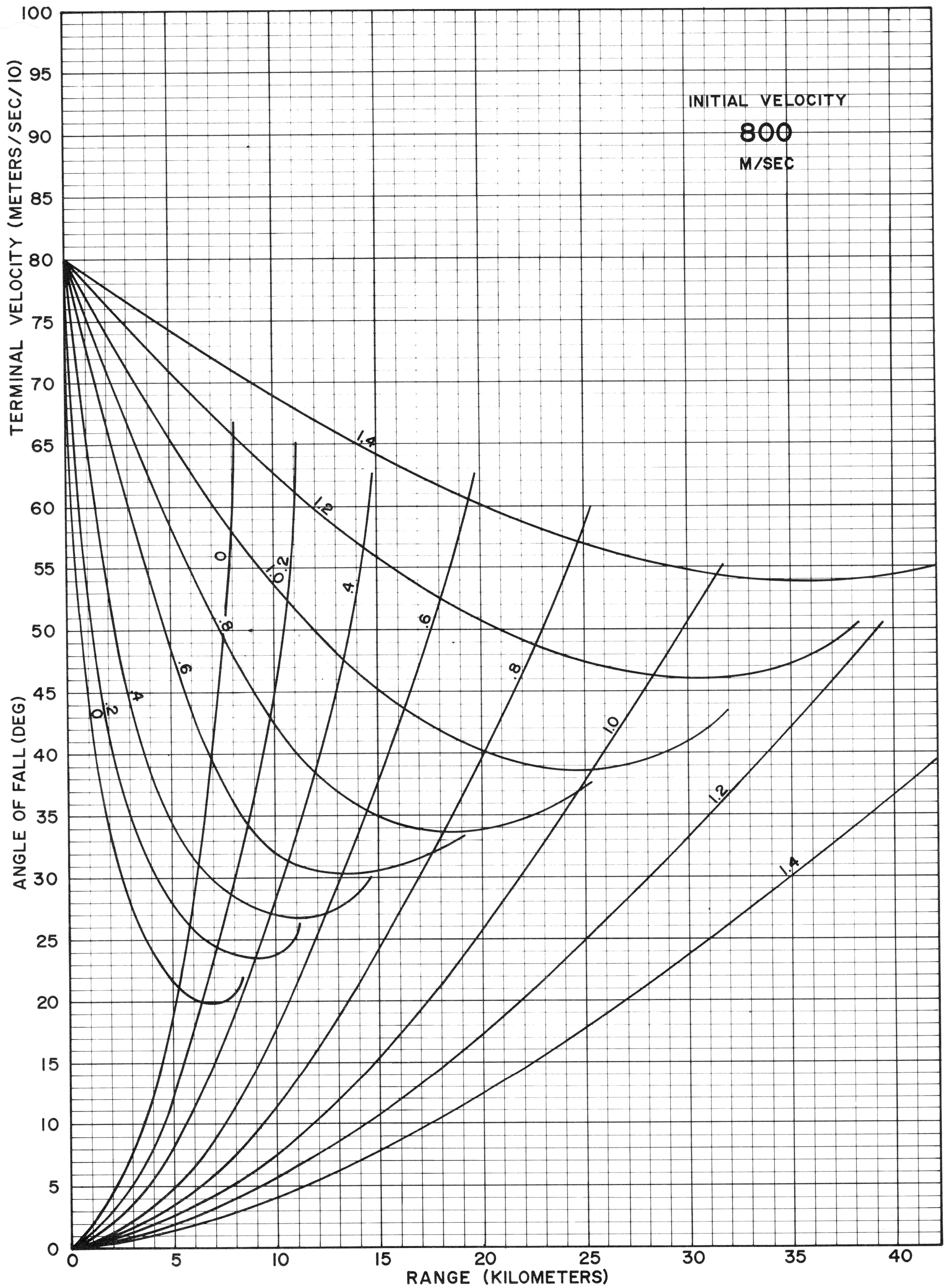
INITIAL VELOCITY
800
M/SEC
TERMINAL VELOCITY (METERS/SEC/10)
ANGLE OF FALL (DEG)
RANGE (KILOMETERS)
0
.2
.4
.6
.8
1.0
1.2
1.4
0 5 10 15 20 25 30 35 40
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

Figure 7

over simplification in places and compression to incomprehensibility in others. If the work is critical, extreme caution should be used until the accuracy of the results can be confirmed through alternative sources. Readers with questions and comments are encouraged to contact the author directly at the address shown at the header of the program listing. I would also appreciate hearing from any I.N.R.O. members who have modified the program to run on other micro-computers, and in languages other than Basic, so that it might be passed on to others who could not otherwise translate it for themselves; and from any member who is fluent on both the Fortran and Basic languages, who might be willing to help translate a number of related programs as yet unavailable in Basic. The program given in this paper can be freely reproduced if used in strictly non-commercial applications. For those readers who might own an Apple computer, I will duplicate the master program on a disk or tape if they supply me the necessary blank media.

Variable List for Various Drag Functions

MACH NO VS KD2 FUNCTION

MACH NO.	KD
.5	.0825
.7	.0825
.8	.0825
.9	.0825
.95	.094
1	.14
1.05	.172
1.1	.175
1.15	.175
1.2	.1725
1.25	.169
1.3	.166
1.35	.1625
1.4	.1605
1.45	.157
1.5	.1535
1.55	.151
1.6	.149
1.65	.146
1.7	.143
1.75	.14
1.8	.138
1.85	.136
1.9	.1335
1.95	.132
2	.129
2.05	.127
2.1	.125
2.15	.123
2.2	.121
2.25	.119
2.3	.1175
2.35	.116
2.4	.1145
2.45	.112
2.5	.111
2.55	.109
2.6	.108
2.65	.106
2.7	.105
2.75	.104
2.8	.103
3.25	.09
3.8	.08
4.15	.075
4.4	.0735

MACH NO VS KD8 FUNCTION

MACH NO.	KD
.5	.0825
.7	.0825
.8	.0825
.9	.0825
.95	.094
1	.14
1.05	.172
1.1	.175
1.15	.175
1.2	.1725
1.25	.169
1.3	.166
1.35	.1625
1.4	.1605
1.45	.157
1.5	.1535
1.55	.151
1.6	.149
1.65	.146
1.7	.143
1.75	.14
1.8	.138
1.85	.136
1.9	.1335
1.95	.132
2	.129
2.05	.127
2.1	.125
2.15	.123
2.2	.121
2.25	.119
2.3	.1175
2.35	.116
2.4	.1145
2.45	.112
2.5	.111
2.55	.109
2.6	.108
2.65	.106
2.7	.105
2.75	.104
2.8	.103
3.25	.09
3.8	.08
4.15	.075
4.4	.0735

MACH NO VS KD (1938 LAW)

MACH NO.	KD
.5	.052
.7	.052
.8	.0538
.9	.0589
.95	.06712
1	.09127
1.05	.11125
1.1	.1227
1.15	.12982
1.2	.13429
1.25	.1368
1.3	.13808
1.35	.13837
1.4	.138074
1.45	.13736
1.5	.1366
1.55	.13554
1.6	.1344
1.65	.1333
1.7	.1319
1.75	.1307
1.8	.1294
1.85	.128
1.9	.1268
1.95	.1253
2	.124
2.05	.1228
2.1	.12155
2.15	.12035
2.2	.119
2.25	.1178
2.5	.1133
2.75	.1085
3	.0975
3.5	.0865

Continued ➧

Variable List for Various Drag Functions—*Continued*

MACH NO VS KD (1940 LAW)

MACH NO.	KD
.5	.085
.7	.085
.8	.085
.9	.085
.95	.085
1	.16
1.05	.1775
1.1	.182
1.15	.181
1.2	.179
1.25	.175
1.3	.17
1.35	.166
1.4	.163
1.45	.16
1.5	.155
1.55	.154
1.6	.151
1.65	.149
1.7	.1455
1.75	.143
1.8	.1405
1.85	.138
1.9	.135
1.95	.133
2	.131
2.05	.129
2.1	.1265
2.15	.1249
2.2	.1225
2.25	.12
2.5	.1125
2.75	.105
3	.0975
3.5	.0865

MACH NO VS KD (ARROW)

MACH NO.	KD
.015	.051
.02	.051
.028	.051
.157	.048
.314	.048
.485	.047
.65	.052
.792	.065
.8667	.08
.934	.095
.994	.109
1.023	.127
1.068	.141
1.098	.149
1.158	.151
1.24	.145
1.315	.136
1.427	.126
1.546	.114
1.673	.103
1.822	.091
2.039	.081
2.255	.075
2.514	.071
2.748	.069
3.211	.065
3.502	.063
3.726	.062
3.927	.06
3.987	.06
0	0
0	0
0	0
0	0

MACH NO VS KD (SPHERE)

MACH NO.	KD
.015	.188
.02	.188
.1	.188
.15	.187
.324	.19
.538	.201
.657	.212
.747	.231
.829	.252
.897	.273
.948	.293
1.023	.317
1.103	.343
1.195	.366
1.44	.388
1.643	.393
1.95	.392
2.3	.386
2.688	.378
3.047	.373
3.323	.369
3.54	.367
3.72	.365
3.965	.363
0	0
0	0
0	0

MACH NO VS KD7 FUNCTION

MACH NO.	KD
.5	.046
.75	.046
.809	.0482
.84	.05
.875	.054
.925	.0625
.94	.0675
1	.146
1.02	.156
1.06	.1595
1.165	.155
1.235	.15
1.35	.1425
1.5	.1345
1.75	.125
2	.1163
2.25	.11
2.5	.1045
2.72	.0998
3	.0944
3.25	.0885
3.5	.0825
4	.072
4.5	.0603

MACH NO VS KD1 (GAVRE) LAW

MACH NO.	KD
.5	.0795
.7	.085
.8	.099
.9	.1362
.95	.161
1	.19
1.05	.213
1.1	.2295
1.15	.242
1.2	.251
1.25	.2555
1.3	.258
1.35	.26
1.4	.2605
1.45	.26
1.5	.258
1.55	.2565
1.6	.254
1.65	.252
1.7	.2495
1.75	.247
1.8	.2445
1.85	.242
1.9	.239
1.95	.236
2	.233
2.05	.231
2.1	.228
2.15	.2255
2.2	.224
2.25	.2215
2.3	.219
2.35	.217
2.4	.215
2.45	.2135
2.5	.212
2.55	.2105
2.6	.2095
2.65	.208
2.7	.207
2.75	.206
2.8	.205
2.85	.2045
2.9	.203
2.95	.2025
3	.202
3.1	.201
3.2	.2

```
]PRINT CHR$(27);"J,100,900,$"

]PRINT CHR$(9);"132N"

]LIST

10  HOME : VTAB 3: HTAB 4: INVERSE : PRINT "MASTER EXTERIOR BALLISTICS PROGRAM"
20  NORMAL
30  PRINT "****************************************"
40  PRINT "   COPYRIGHT (C) (1983) W. J. JURENS"
50  PRINT "             62 FIDLER AVENUE"
60  PRINT "      WINNIPEG, MANITOBA, CANADA"
70  PRINT "                 R3J 2R7"
80  PRINT "              PH. 204-837-3125"
90  PRINT "****************************************"
100  PRINT
110  PRINT "  PRESS ANY KEY TO LOAD THE PROGRAM": PRINT
120  GET A$
130  DIM M(50): DIM K(50): REM   DIMENSION VARIABLES FOR M=MACH NO. AND K= DRAG COEFFICIENT FROM GRAPH OF AMERICAN KD8 FUNCTIO
     N
140 M(0) = .3:K(0) = .0825:M(1) = .5:K(1) = .0825:M(2) = .7:K(2) = .0825:M(3) = .8:K(3) = .0825:M(4) = .9:K(4) = .0825:M(5) =
     .95:K(5) = .094
150 M(6) = 1:K(6) = .140:M(7) = 1.05:K(7) = .172:M(8) = 1.1:K(8) = .175:M(9) = 1.15:K(9) = .175:M(10) = 1.2:K(10) = .1725:M(11
     ) = 1.25:K(11) = .1698
160 M(12) = 1.30:K(12) = .166:M(13) = 1.35:K(13) = .1625:M(14) = 1.40:K(14) = .1605:M(15) = 1.45:K(15) = .157
170 M(16) = 1.5:K(16) = .1535:M(17) = 1.55:K(17) = .151:M(18) = 1.6:K(18) = .149:M(19) = 1.65:K(19) = .146:M(20) = 1.7:K(20) =
     .143:M(21) = 1.75:K(21) = .14
180 M(22) = 1.8:K(22) = .138:M(23) = 1.85:K(23) = .136:M(24) = 1.9:K(24) = .1335:M(25) = 1.95:K(25) = .132:M(26) = 2:K(26) = .
     129:M(27) = 2.05:K(27) = .127:M(28) = 2.10:K(28) = .125:M(29) = 2.15:K(29) = .123:M(30) = 2.2:K(30) = .121
190 M(31) = 2.25:K(31) = .119:M(32) = 2.3:K(32) = .1175:M(33) = 2.35:K(33) = .116:M(34) = 2.4:K(34) = .1145:M(35) = 2.45:K(35)
      = .112:M(36) = 2.5:K(36) = .111
200 M(41) = 2.75:K(41) = .104:M(42) = 2.8:K(42) = .103:M(43) = 3.25:K(43) = .09:M(44) = 3.8:K(44) = .08:M(45) = 4.15:K(45) = .
     075:M(46) = 4.40:K(46) = .0735
210  PRINT
220 RD = 57.295779513: REM  DEG/RAD CONVERSION
230 DF = 1.2250: REM   DENSITY IN KG/M^3
240 Z4 = 1.34279408E - 18:Z3 =  - 9.87941429E - 14:Z2 = 3.90848966E - 9:Z1 =  - 9.69888125E - 5: REM    POLYNOMIAL TERMS FOR I
     CAO STD ATMOSPHERE
250  INPUT " INPUT INITIAL VELOCITY (M/SEC) ";VO
260  PRINT
270  INPUT " INPUT ANGLE OF DEPARTURE (DEG) ";LD
280  PRINT
290  INPUT " INPUT PROJ. CALIBER (MM) ";DS
300 D = DS / 10: REM   CAL MUST BE IN CENTIMETERS FOR FURTHER CALCS
310  PRINT
320  INPUT " INPUT PROJ. MASS (KG) ";M
330  PRINT
340  INPUT " INPUT FORM FACTOR ";FF
350  PRINT
360  INPUT " INPUT INTEGRATION INTERVAL (SEC) ";I
370  PRINT
380  INPUT " AUTO-SET INTERVALS? (Y/N) ";IC$: PRINT
390  IF IC$ = "Y" THEN  INPUT " INPUT ERROR TOLERANCE (M/SEC) ";ET: PRINT
400  INPUT " INPUT AIR DENSITY FACTOR ";AD
410  PRINT
420  INPUT " INPUT ALTITUDE (METERS) ";AO
430  PRINT
440  PRINT " INPUT CHOICE OF DENSITY FUNCTIONS:"
450  PRINT "    U.S. PRE-1945 STD= 1"
460  PRINT "    BRITISH STANDARD = 2"
470  PRINT "    I.C.A.O STANDARD = 3"
480  INPUT PW
490  HOME : VTAB 3: HTAB 4: INVERSE : PRINT "SUMMARY OF INITIAL CONDITIONS"
500  NORMAL : PRINT
510  PRINT "  INITIAL VELOCITY = ";VO;" METERS/SEC."
520  PRINT "  ANGLE OF DEPARTURE = ";LD;" DEGREES"
530  PRINT "  PROJECTILE CALIBER = ";DS;" MM"
540  PRINT "  PROJECTILE WEIGHT  = ";M;" KG"
550  PRINT "  FORM FACTOR        = ";FF
560  PRINT "  AIR DENSITY FACTOR = ";AD
570  PRINT "  INTEGRATION INTERVAL=";I;" SEC"
580  PRINT "  INITIAL ALTITUDE   = ";AO;" METERS"
590  PRINT "  ";X$
600  IF PW = 1 THEN  PRINT "  U.S. PRE-1945 DENSITY FUNCTION"
610  IF PW = 2 THEN  PRINT "  BRITISH STD. DENSITY FUNCTION"
620  IF PW = 3 THEN  PRINT "  I.C.A.O. DENSITY FUNCTION"
630  PRINT : PRINT : INVERSE : PRINT "PRESS 'G' TO GO"
```

```
640 NORMAL : PRINT "(ANY OTHER KEY WILL RECYCLE)": PRINT
650 GET PP$
660 IF PP$ < > "G" THEN HOME : GOTO 180
670 HOME : VTAB 3: HTAB 4: INVERSE : PRINT "TABULAR INTEGRATION OF TRAJECTORY": NORMAL : PRINT
680 PRINT "TIME   RANGE   HEIGHT   ANGLE   VELOCITY"
690 PRINT "SEC.   METERS  METERS    DEG    M/SECOND"
700 PRINT TT TAB( 8) INT (10 * RG + .5) / 10 TAB( 16) INT (10 * AO + .5) / 10 TAB( 24) INT (1000 * (LD) + .5) / 1000 TAB( 33)
    INT (1000 * VO + .5) / 1000
710 REM :ABOVE LINE TO PRINT INITIAL CONDITIONS
720 LD = LD / RD: REM  CONVERT DEGREES TO RADIANS FOR CALCS
730 REM **BEGIN NUMERICAL INTEGRATION PROCEDURE**
740 C = M / (FF * AD * D ^ 2): REM  BALLISTIC COEFFICIENT
750 IF TT = 0 THEN  GOTO 830
760 PRINT TT TAB( 8) INT (10 * RG + .5) / 10 TAB( 16) INT (10 * AO + .5) / 10 TAB( 24) INT (1000 * (L3 * RD) + .5) / 1000 TAB(
    33) INT (1000 * VO + .5) / 1000
770 REM  :ABOVE LINE TO PRINT TYPICAL CONDITIONS
780 P3 = P2:P2 = P1:P1 = RG: REM  RANGE ARRAY
790 Q3 = Q2:Q2 = Q1:Q1 = VO: REM  VELOCITY ARRAY
800 S3 = S2:S2 = S1:S1 = L3: REM  ANGLE ARRAY
810 T3 = T2:T2 = T1:T1 = TT: REM  TIME ARRAY
820 U3 = U2:U2 = U1:U1 = AO: REM  ALTITIUDE ARRAY
830 IF AO < 0 THEN  GOSUB 1840
840 YE = YE + 1
850 IF YE = 5 THEN  PRINT :YE = 0
860 REM :ABOVE LINES TO FORMAT PRINTOUT
870 Y = AO: REM  SET UP FOR DENSITY
880 G = 9.80665 - (.0000030665 * AO): REM  ACCEL OF GRAVITY
890 CS = 344 - (.004 * AO): REM  SOUND SPEED (M/SEC)
900 XO = VO *  COS (LD): REM  HORZ COMPONENT
910 YO = VO *  SIN (LD): REM  VERT COMPONENT
920 REM :COMPUTE RETARDATION
930 KK = VO / CS: REM  MACH NO.
940 FOR X = 1 TO 50
950 IF KK =  < M(X) THEN  GOTO 970
960 NEXT X
970 KD = (KK - M(X - 1)) / (M(X) - M(X - 1)) * (K(X) - K(X - 1)) + K(X - 1): REM  LINEAR INTERP. ON DRAG FUNCTION ARRAY
980 RO = KD * (DF / 10000) * VO ^ 2: REM  RETARDATION IN M/SEC^2
990 Y = A1 + AO
1000 IF PW = 1 THEN  GOSUB 1770
1010 IF PW = 2 THEN  GOSUB 1790
1020 IF PW = 3 THEN  GOSUB 1810
1030 DO = LL
1040 EO = RO / (C / DO): REM  ACTUAL RETARDATION
1050 HO = EO *  COS (LD): REM  1ST GUESS AT HORIZ COMPONENT OF RETARDATION
1060 JO = (EO *  SIN (LD) + G): REM  1ST GUESS AT VERT COMPONENT FO RETARDATION
1070 X1 = XO - (HO * I): REM  1ST PRED. OF HORIZ VEL
1080 Y1 = YO - (JO * I): REM  1ST PRED. OF VERT VEL
1090 V1 =  SQR (X1 ^ 2 + Y1 ^ 2)
1100 L1 =  ATN (Y1 / X1): REM  1ST PRED. OF FINAL ANGLE
1110 REM ** BEGIN SECOND PREDICTION **
1120 M1 = (YO + Y1) / 2: REM  ESTIMATE OF MEAN VERT VELOCITY
1130 A1 = (M1 * I): REM  EST. ALTITUDE AT END OF INTERVAL
1140 Y = A1 + AO: REM   SET UP TO COMPUTE DENSITY FOR ALTITUDE
1150 IF PW = 1 THEN  GOSUB 1770
1160 IF PW = 2 THEN  GOSUB 1790
1170 IF PW = 3 THEN  GOSUB 1810
1180 D1 = LL
1190 KK = V1 / CS: REM   MACH NO.
1200 FOR X = 1 TO 50
1210 IF KK =  < M(X) THEN  GOTO 1230
1220 NEXT X
1230 KD = (KK - M(X - 1)) / (M(X) - M(X - 1)) * (K(X) - K(X - 1)) + K(X - 1): REM   LINEAR INTERP. ON DRAG FUNCTION ARRAY
1240 R1 = KD * (DF / 10000) * V1 ^ 2: REM   RETARDATION IN M/SEC^2
1250 E1 = R1 / (C / D1): REM  ACTUAL RETARDATION
1260 H1 = E1 *  COS (L1): REM  2ND GUESS AT HORIZ RET. COMPONENT
1270 J1 = (E1 *  SIN (L1) + G): REM   2ND GUESS AT VERT RET. COMPONENT
1280 H2 = (HO + H1) / 2: REM  3RD EST OF HORIZ RET COMPONENT
1290 J2 = (JO + J1) / 2: REM  3RD EST OF VERT RET COMPONENT
1300 X2 = XO - (H2 * I): REM  2ND PRED OF HORIZ VEL
1310 Y2 = YO - (J2 * I): REM  2ND PRED OF VERT VEL
1320 V2 =  SQR (X2 ^ 2 + Y2 ^ 2): REM  2ND PRED OF FINAL VELOCITY
1330 L2 =  ATN (Y2 / X2): REM  2ND PRED OF FINAL ANGLE
1340 REM ** BEGIN THIRD PREDICTION **
1350 M2 = (YO + Y2) / 2: REM  2ND EST OF MEAN VERT VEL
1360 A2 = M2 * I: REM  EST ALT AT END OF INTERVAL
1370 Y = A2 + AO: REM   SET UP FOR DENSITY CALC
1380 IF PW = 1 THEN  GOSUB 1770
1390 IF PW = 2 THEN  GOSUB 1790
1400 IF PW = 3 THEN  GOSUB 1810
1410 D2 = LL
```

```
1420 KK = V2 / CS: REM  MACH NO
1430  FOR X = 1 TO 50
1440  IF KK = < M(X) THEN  GOTO 1460
1450  NEXT X
1460 KD = (KK - M(X - 1)) / (M(X) - M(X - 1)) * (K(X) - K(X - 1)) + K(X - 1): REM    LINEAR INTERP. ON DRAG FUNCTION ARRAY
1470 R2 = KD * (DF / 10000) * V2 ^ 2: REM   RETARDATION IN M/SEC^2
1480 E2 = R2 / (C / D2): REM   ACTUAL RETARDATION
1490 H3 = E2 *  COS (L2): REM   4TH GUESS AT HORIZ RET. COMPONENT
1500 J3 = (E2 *  SIN (L2) + G): REM    4TH GUESS AT VERT RET. COMPONENT
1510 H4 = (H0 + H3) / 2: REM  5TH EST OF HORIZ RET. COMPONENT
1520 J4 = (J0 + J3) / 2: REM  5TH EST OF VERT RET. COMPONENT
1530 X3 = X0 - (H4 * I): REM  3RD PRED OF HORIZ VEL
1540 Y3 = Y0 - (J4 * I): REM  3RD PRED OF VERT VEL
1550 V3 =  SQR (X3 ^ 2 + Y3 ^ 2): REM  3RD PRED OF FINAL VELOCITY
1560 L3 =  ATN (Y3 / X3): REM  3RD PRED OF FINAL ANGLE
1570 M3 = (Y0 + Y3) / 2: REM  3RD EST OF MEAN VERT VEL
1580 M4 = (X0 + X3) / 2: REM  3RD EST OF MEAN HORIZ VEL
1590 FH = M4 * I: REM   FINAL HORIZ POSN
1600 FV = M3 * I: REM   FINAL VERT POSN
1610 LI = I: REM  PUT INTERVAL IN SCRATCH VARIABLE *BEFORE* CHANGING VIA AUTO SET
1620  REM :** END OF 3RD PREDICTION **
1630  REM :DO AUTO SET OF INTERVAL
1640  IF IC$ < > "Y" THEN  GOTO 1680
1650  IF  ABS (V3 - V2) < .5 * ET THEN I = I * 1.2
1660  IF  ABS (V3 - V2) > .8 * ET THEN I = I * .8
1670  IF  ABS (V3 - V2) > ET THEN  INVERSE : PRINT "ERROR TOLERANCE EXCEEDED": NORMAL :I = I / 2: REM  IF ERROR EXCEEDED WARN
     AND REDUCE INTERVAL
1680  REM :** SETUP TO LOOP BACK **
1690  IF VE > ME THEN ME = VE: REM  LINE TO FIND LARGEST VEL ERROR
1700 VE =  ABS (V2 - V3): REM  ERROR IN FINAL VELOCITY PREDICTIONS
1710 V0 = V3
1720 LD = L3
1730 A0 = A0 + FV: REM   FINAL ALTITUDE
1740 RG = RG + FH: REM  SUM TOTAL RANGE
1750 TT =  INT (1000 * (TT + LD + .5) / 1000: REM   SUM INTEGRATION INTERVALS FOR TOTAL TIME & CORRECT INTERPRETER ROUNDOFF E
     RROR
1760  GOTO 740
1770 LL = 10 ^  - (.000045 * Y)
1780  RETURN
1790 LL = .1 ^ (.141 * (Y / 3048))
1800  RETURN
1810 LL = Z4 * Y ^ 4 + Z3 * Y ^ 3 + Z2 * Y ^ 2 + Z1 * Y + 1
1820  RETURN
1830  REM  BEGIN FINAL INTERPOLATION ROUTINE
1840 Y(1) = P3:Y(2) = P2:Y(3) = P1
1850 X(1) = U3:X(2) = U2:X(3) = U1
1860  GOSUB 2050
1870 RL = B
1880 Y(1) = Q3:Y(2) = Q2:Y(3) = Q1
1890  GOSUB 2050
1900 SL = B
1910 Y(1) = S3:Y(2) = S2:Y(3) = S1
1920  GOSUB 2050
1930 AL = B
1940 Y(1) = T3:Y(2) = T2:Y(3) = T1
1950  GOSUB 2050
1960 TL = B
1970  PRINT : PRINT : HTAB 4: INVERSE : PRINT "INTERPOLATED TERMINAL CONDITIONS": NORMAL : PRINT
1980  PRINT "RANGE = "; INT (RL);" METERS"
1990  PRINT "HEIGHT = 0 METERS"
2000  PRINT "VELOCITY = "; INT (10 * SL + .5) / 10;" METERS/SEC."
2010  PRINT "ANGLE OF FALL = ";AL * RD;" DEGREES"
2020  PRINT "TIME = "; INT (TL * 100 + .5) / 100;" SECONDS"
2030  PRINT : HTAB 10: PRINT "FINISHED WITH PROBLEM": END
2040  REM  BEGIN 3 POINT ROUTINE
2050 P = 3
2060 B = 0
2070 A = 0
2080  FOR J = 1 TO P
2090 T = 1
2100  FOR I = 1 TO P
2110  IF I = J THEN 2130
2120 T = T * (A - X(I)) / (X(J) - X(I))
2130  NEXT I
2140 B = B + T * Y(J)
2150  NEXT J
2160  RETURN
```

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W. J. JURENS was born in 1947 in Winnipeg, Canada and has lived there all his life. After graduation from a local technical school, worked eight years in the transportation and aerospace industry as a draftsman and an engineering aide, while attending night school to gain a Bachelor of Education degree. He has been employed for the past eight years as a Vocational Drafting Instructor with a local school division.

He is married with two children. He first became interested in ships and ship design while in grade school and, apart from a hiatus of several years while attending the University, he has been studying the subject more or less constantly on a part time basis all that time. Mr. Jurens has been a member of INRO since 1964. Nathan Okun and he are currently in the process of co-authoring a book length manuscript on the closely related subjects of ballistics and armor penetration—a project which will likely take them another two or three years to complete.